DOCUMENT RESUME

ED 080 362 SE 016 615

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TITLE	Mathematics [Sahuarita High School Career Curriculum Project].
INSTITUTION	Sahuarita High School District 130, Ariz.
PUB DATE	[73]
NOTE	294p.
EDRS PRICE	MF-$0.65 HC-$9.87
DESCRIPTORS	Activity Units; Curriculum Guides; *Instructional Materials; Logic; *Mathematics Education; *Practical Mathematics; *Secondary School Mathematics; Teacher Developed Materials; Trigonometry; *Units of Study (Subject Fields)

ABSTRACT

This is a collection of units prepared by high school mathematics teachers to be used as a "career curriculum" guide. Each unit contains a statement of objectives, a discussion of the content, activity sheets, and exercises. The volume is organized into clusters so that an individual may choose the parts suited to individual needs. Major topics included are logic; ratio and proportion; volume; Boolean algebra; trigonometry, with emphasis on use of tables and with applications in the machine trades and in surveying; and vector mechanics, primarily determination of forces by the parallelogram method and by resolution into components and also including the use of dot product, cross product, and determinants. Related volumes in the series are SE 016 616 through SE 016 618. (LS)

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MATH

MADEHEIM

SE 016 615

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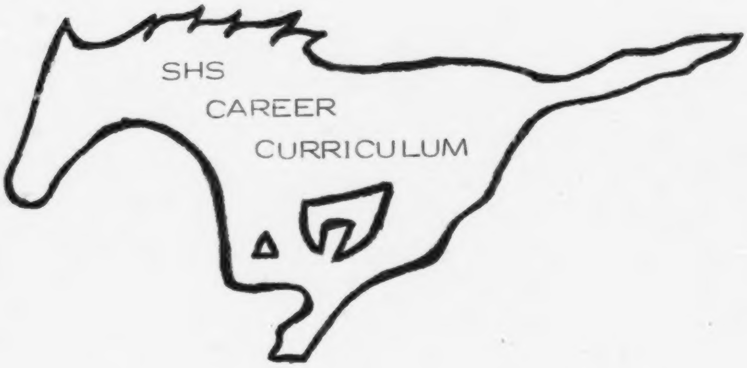


COURSE TITLE: MATH

PACKAGE TITLE: "OH NO!!" "I QUIT. I REFUSE!!"
"IT'S IMPOSSIBLE."

BY

JIM MADEHEIM

MATH

BEHAVIORAL OBJECTIVES

1. Given a logic problem that can be solved systematically, you will be able to set up some kind of procedure to aid your finding the correct solution, and then find the correct solution, and then find the correct solution for at least half the problem.

2. Given problems applicable to the machine trades that involve the use of trig to find their solution, you will be able to solve the problems with four digit accuracy at least 75% of the time.

3. Given a problem that asks for the resultant of a set of forces (in a plane), you will be able to find this resultant using the parallelogram and vector methods, both, with 60% accuracy. (You will also be given a grade for specific homework problems that are asked for. All tests are open book).

4. Given some problems to solve involving the finding of the resultant of some forces in space,,you will be able to find these resultants in 60% of the problems. (All tests are open book. You will also be given a grade for specific homework problems that are asked for).

5. Given some problems to solve involving the findings of moments, you will be able to find the moments by using the cross product, (for 3-D problems you must use determinants), with 60% accuracy.

6. Given some problems that require you to use the mixed triple products of three vectors, you will be able to solve those problems with 60% accuracy. (Open book test).

BEHAVIORAL OBJECTIVES (Cont.)

7. Given access to reference materials, and given a problem to solve that involves finding the volume of an object pertaining to the machine trades, you will be able to find that volume with four digit accuracy, 75% of the time.

8. Given the number of teeth or the dimensions of a set of gears, you will be able to find the ratios of all the gears to each other with at least 75% accuracy

9. Given the necessary information to set up a proportion between gears or pulleys, you will be able to find the unknown quantity of the proportion with at least 75% accuracy.

10. Given any two sides of a right triangle, you will be able to find the third side with two decimal place accuracy.

Given the distance "a" and the distance "b" as in the picture below, representing two legs of a right triangle, and the use of the electronic calculator and square root tables, you will be able to find the length of the cable with an accuracy of one foot.

11. Given any anle between 0 and 90°you will be able to look up its tangent in a trig table with 100% accuracy.

Given the tangent of an angle, you will be able to name the nearest whole angle with 1°accuracy.

Given a distance to measure that requires the use of the tangent formula, you will be able to find the distance wanted to the nearest whole number using the tangent formila, trig tables, and calculator.

12. Given any angle between 0° and 90°, you will be able to look up its sine exactly.

Given the sine of an angle, you will be able to find the angle with one degree accuracy.

BEHAVIORAL OBJECTIVES (Cont.)

Given a situation similar to the one pictured above, where the side opposite a given angle is to be found and the hypotenuse is known, you will be able to find the distance asked for by using the sine formula, with whole number accuracy.

13. Given the air speed and angle of elevation of an airplane you will be able to calculate its ground speed with whole number accuracy.

14. Given different situations involving indirect measurement by use of the sine, cosine, and tangent formulas, you will be able to pick which of the three formulas, to use and find the distance with whole number accuracy.

15. Given a problem to solve that requires the use of the Law of Cosines, you will be able to set up the equation and solve the problem with 3 decimal place accuracy, with the use of the calculator and trig tables.

16. Given a Boolean expression, you will be able to simplify it, if possible, using the postulates and theorems of Boolean algebra and then draw a schematic of the expression, with 80% accuracy.

17. Given a complicated series of statements that can be expressed using boolean algebra statements, you will be able to change the statements to algebraic expressions and simplify them, and draw a circuit picturing the simplification and test it for validity. Your drawing will be graded and corrected before you test for validity. You must set up and answer correctly 80% of the problems.

(You will not be told your grade on the pre-test, unless you pass , nor will you be able to look at it until this package is completed.)

Rationale:

Intuition is fine, sometimes. Guess work is quick, except when you guess wrong. A systematic approach to solving a problem is usually the best way to be sure your problem gets solved correctly.

This unit develops several "systematic approaches" for solving those familiar logic problems that always seem impossible at first glance.

Behavioral Objectives:

Given a logic problem that can be solved systematically, you will be able to set up some kind of procedure to aid your finding the correct solution, and then find the correct solution for at least half the problem.

Pre-evaluation:

1. Nine men - Brown, White, Adams, Miller, Gren, Hunter, Knight, Jones, and Smith - play the several positions on a baseball team. (The battery consists of the pitcher and the catcher; the infield consists of the first, second, and third basemen and the shortstop; the outfield consists of the right, left, and center fielders). Determine from the following data, the position played by each man.

   a. Smith and Brown each won $10 playing poker with the pitcher.
   b. Hunter was taller than Knight and shorter than White, but each of these weighed more than the first baseman.
   c. The third baseman lives across the corridor from Jones in the same apartment house.
   d. Miller and the outfielders play bridge in their spare time.
   e. White, Miller, and Brown, the right fielder, and the center fielder were bachelors; the rest were married.
   f. Of adams and Knight, one played outfielder position.
   g. The right fielder was shorter than the center fielder.
   h. The third baseman was brother to the pitcher's wife.
   i. Green was taller than the infielders and the battery, except for Jones, Smith, and Adams.
   j. The third baseman, the shortstop, and Hunter made #150 each speculating in U.S. Steel.
   k. The second baseman was engaged to Miller's wife.

l. The second baseman beat JOnes, Brown, Hunter and the catcher at cards.

m. Adam lives in the same house as his own sister but dislikes the catcher.

n. Adams, Brown, and the shortstop lost $200 each speculating in copper.

o. The catcher had three daughters, the third baseman had two sons, but Green was being sued for divorce.

2. Three men - A, B and C - are aware that all three of them are "perfect logicians" who can instantly deduce all the consequences of a given set of premises. There are four red and four green stamps available. The men are blindfolded and two stamps are pasted on each man's forehead. The blindfolds are removed. A, B and C are asked in turn: "Do you know the color of your stamps?" Each says: "No." The question is then asked of A once more. He again says: "No." B is now asked the question, and replies: "Yes." What are the colors of B's stamps?

Information Sources:

1. Read Data Brief #1 "The Hats"
2. Read Data Brief #2 "Who Dunnit?"
3. Read Data Brief #3 "Buddies"
4. Read Data Brief #4 "Conglomeration"

Data Brief #1

Of the three prisoners in a certain jail, one had normal vision, the second had only one eye, and the third was totally blind. All were of at least average intelligence. The jailer told the prisoners that from three white hats and two red hats he would select three and put them on the prisoner's heads. Each was prevented from seeing what color hat was placed on his own head. They were brought together, and the jailer offered freedom to the prisoner with normal vision if he could tell what color hat was on his head. The prisoner confessed that he couldn't tell. Next the jailer offered freedom to the prisoner with only one eye if he could tell what color hat was on his head. The second prisoner confessed that he couldn't tell. The jailer did not bother making the offer to the blind prisoner but agreed to extend the same terms to him when he made the request. The blind prisoner then similed broadly and said:

"I do not need to have my sight;
From what my friends with eyes have said
I clearly see my hat is ____________________!"

Below is a list of all possible ways a hat can be chosen

	2 eyes	1 eye	blind
1	w	w	w
2	w	r	r
3	r	w	r
4	r	r	w
5	w	w	r
6	w	r	w
7	r	w	w

Since only two red hats were used at the most, if two eyes saw two red hats he would know that he was white and would have said so. But he didn't know. Therefore # 2 above is not possible.

By the same reasoning 1 eye couldn't tell he was white so he must not have seen two red hats. So #3 is out.

Also, 1 eye knows he and blind don't both have red hats, because two eye didn't say anything.

Therefore if 1 eye saw a red hat on blind, he would know that he himself must be white, because they both weren't red. But he didn't know this since he didn't speak. Therefore 1 eye couldn't have seen a red hat on the blind man.

Therefore the blind man, who reasoned this all out, must be white.

Data Brief #2

Benno Torelli, genial host at Hamtramcks most exclusive nightclub, was shot and killed by a racketeer gang because he fell behind in his protection payments. After considerable effort on the part of the police, five men were brought before the District Attorney, who asked them what they had to say for themselves. Each of the men made three, two true and one false. Their statements were:

Lefty:
1. I did not kill Torelli.
2. I never owned a revolver.
3. Spike did it.

Red:
1. I did not kill Torelli.
2. I never owned a revolver.
3. The other guys are passing the buck.

Spike:
1. I am innocent.
2. Butch is the guilty man.
3. Lefty lied when he said I did it.

Dopey:
1. I am innocent.
2. I never saw Butch before.
3. Spike is guilty.

Butch:
1. I did not kill Torelli.
2. Red is the guilty man.
3. Dopey and I are old pals.

Who Dunnit?

Start this way. If Butch #1 is false, then Butch #2 is true. But this would say that both Butch and Red did the killing, which can't be. Therefore your original assumption is false, and it must be true that Butch did not kill Torelli.

Use the same type of reasoning on Dopey #1 and #3.

Use it also on Lefty #1 and #3.

Who dunnit?

Data Brief #3

The sentence in the problem below are taken out of their paragraph form to make it easier to demonstrate how to solve this problem.

Five men who were buddies in the last war are having a reunion. They are White, Brown, Peters, Harper, and Nash, who by occupation are printer, writer, barber, neurologist, and heating-contractor, though not necessarily in that order. By coincidnece they live in the cities of White Plains, Brownsville, Petersburg, Harper's Ferry, and Nashville, though not necessarily in that order. But,

1. no man lives in the city having a similar name to his,

2. nor does the name of his occupation have the same initial as his name

3. or the name of the city in which he lives.

4. The barber doesn't live in Petersburg,

5. and Brown is neither heating-contractor nor printer

6. nor does he live in Petersburg or Harper's Ferry.

7. Mr. Harper lives in Nashville and is neither barber or writer.

8. White is not a resident of Brownsville,

9. nor is Nash, who is not a barber, nor a heating-contractor.

What city does Nash live in. (See the next page for this problem written without numbered sentences.)

Buddies

Five men who were buddies in the late war are having a reunion. They are White, Brown, Peters, Harper, and Nash, who by occupation are printer, writer, barber, neurologist, and heating-contractor. By coincidence, they live in the cities of White Plains, Brownsville, Petersburg, Harper's Ferry, and Nashville, but no man lives in the city having a name similar to his, nor does the name of his occupation have the same initial as his name or the name of the city in which he lives.

The barber doesn't live in Petersburg, and Brown is neither heating-contractor nor printer--nor does he live in Petersburg or Harper's Ferry. Mr. Harper lives in Nashville and is neither barber nor writer. White is not a resident of Brownsville, nor is Nash, who is not a barber, nor a heating-contractor.

If you have only the information given above, can you determine the name of the city in which Nash resides?

Buddies

	White-Plains	Browns-Ville	Peters-Burg	Harper's Ferry	Nash-ville
White		W B H P N	W B H P N	W B H P N	W B H P N
Brown	W B H P N		W B H P N	W B H P N	W B H P N
Peters	W B H P N	W B H P N		W B H P N	W B H P N
Harper	W B H P N	W B H P N	W B H P N		W B H P N
Nash	W B H P N	W B H P N	W B H P N	W B H P N	

Due to sentence #1, the following boxes are crossed out.

Buddies

	White-plains	Browns-ville	Peters-burg	Harper's Ferry	Nash-ville
White		W B H P N	W B H P N	W B H P N	[redacted]
Brown	W B H P N		W B H P N	W B H P N	[redacted]
Peters	W B H P N	W B H P N		W B H P N	[redacted]
Harper	[redacted]	[redacted]	[redacted]		[redacted] [redacted] H P N
Nash	W B H P N	W B H P N	W B H P N	W B H P N	

Due to sentence #7, the following additional boxes are crossed out.

Buddies

	White-plains	Browns-ville	Peters-burg	Harper's Ferry	Nash-ville
White		[redacted]	W B H P N	W B H P N	
Brown	W B H P N		W B H P N	W B H P N	
Peters	W B H P N	W B H P N		W B H P N	
Harper					H P N
Nash	W B H P N	W B H P N	W B H P N	W B H P N	

Due to #8 the following box is crossed out.

Buddies

	White-plains	Browns-ville	Peters-burg	Harper's Ferry	Nash-ville
White			W B H P N	W B H P N	
Brown	W B H P N		W B H P N	W B H P N	
Peters	W B H P N	W B H P N		W B H P N	
Harper					H P N
Nash	W [illegible] [illegible] P N	[illegible]	W [illegible] [illegible] P N	W [illegible] [illegible] P N	

The following portions are crossed out due to #9.

Buddies

	White-plains	Browns-ville	Peters-burg	Harper's Ferry	Nash-ville
White			[redacted] B H P N	[redacted] B H P N	
Brown	W [redacted] H P N		W [redacted] H P N	W [redacted] H P N	
Peters	W B H [redacted] N	W B H [redacted] N		W B H [redacted] N	
Harper					[redacted] P N
Nash	W P [redacted]		W P [redacted]	W P [redacted]	

Due to sentence #2, the following items are crossed out.

Buddies

	White-plains	Browns-ville	Peters-burg	Harper's Ferry	Nash-ville
White			B H [illegible] N	B [illegible] [illegible] N	
Brown	[illegible] H P N		W [illegible] P N	W [illegible] P N	
Peters	[illegible] B H N	W [illegible] H N		W B [illegible] N	
Harper					P [illegible]
Nash	[illegible] P		W [illegible]	W P	

Due to #3 the following items are crossed out.

Because of this last information, you can see that Harper is a printer, and therefore no one else can be a printer. See if you can finish it from here.

Data Brief #4

The mixture of problems in activity #4 can be solved using many different approaches. Some of these approaches have already been discussed. In every case possible, set up a table to aid your finding the solution.

Activity #1

Three men go by turn into a dark closet where hang five hats, three red and two blue. Out they came, each man forbidden to look at his own hat, but permitted to look at the hats of the others in an effort to tell the color of his own. A glances at B and C, and says, "I don't know what color hat I have on." B, who is equally intelligent, looks around and says, "Nor do I know what color hat I have on." What color hat was C wearing and hat was C wearing and how did he figure it out?

Three men are blindfolded and told that either a red or green hat will be placed on each of their heads. After this is done, the blindfolds are removed; the men are asked to raise a hand if they see a red hat, and to leave the room as soon as they are sure of the color of their own hat. All three hats happen to be red, so all three men raise a hand. Several minutes go by until one of them who is more astute than the others, leaves the room. How did he deduce the color of his hat?

Activity #2

Daniel Kilraine was killed on a lonely road, two miles from Pontiac at 3:30 a.m., March 17, 1952. Otto, Curly, Slim, Mickey, and the Kid were arrested a week later in Detroit and questioned. Each of the five made four statements, three of which were true and one of which was false. One of these men killed Kilraine. Who dunnit? Their statements were:

Otto: I was in chicago when Kilraine was murdered.
I never killed anyone.
The Kid is the guilty man.
Mickey and I are pals.

Curly: I din not kill Kilraine.
I never owned a revolver in my life.
The Kid knows me.
I was in Detroit the night of March 17.

Slim: Curly lied when he said he never owned a revolver.
The murder was committed on St. Patrick's Day.
Otto was in Chicago at this time.
One of us is guilty.

Mickey: I did not kill Kilraine.
The Kid never has been in Pontiac.
I never saw Otto before.
Curly was in Detroit with me on the night of March 17.

The Kid: I did not kill Kilraine.
I have never been in Pontiac.
I never saw Curly before.
Otto lied when he said I am guilty.

Activity #3

BRAIN TEASER

HERE IS AN INTERESTING ONE WHICH WILL TEST YOUR LOGICAL ABILITY. ALL THE FACTS NEEDED TO ANSWER THE QUESTIONS POSED ARE CONTAINED IN STATEMENTS 1-15 BELOW.

1. There are five houses, each of a different color and inhabited by men of different nationalities, with different pets, drinks and cigarettes.
2. The Englishman lives in the red house.
3. The Spaniard owns the dog.
4. Coffee is drink in the green house.
5. The ukrainian drinks tea.
6. The green house is immediately to the right of the ivory house.
7. The Old Gold smoker owns snails.
8. Kools are smoked in the yellow house.
9. Milk is drunk in the middle house.
10. The Norwegian lives in the first house on the left.
11. The man who smokes Chesterfields lives in the house next to the man with the fox.
12. Kools are smoked in the house next to the house where the horse is kept.
13. The Lucky Strike smoker drinks orange juice.
14. The Japanese smokes Parliaments.
15. The Norwegian lives next to the blue house.

Now, who drinks water? Who owns the zebra?

SEE THE NEXT PAGE FOR AN AID TO THIS PROBLEM.

	Color	Drink	Pet	Cig	Country
1	Y B I G R	W T M C O	Z H F D S	K C P O L	N J U S E
2	Y B I G R	W T M C O	Z H F D S	K C P O L	N J U S E
3	Y B I G R	W T M C O	Z H F D S	K C P O L	N J U S E
4	Y B I G R	W T M C O	Z H F D S	K C P O L	N J U S E
5	Y B I G R	W T M C O	Z H F D S	K C P O L	N J U S E

LOANS

The members of a small loan company are Mr. Black, Mr. White, Mrs. Coffee, Miss Ambrose, Mr. Kelly, and Miss Earnshaw. The positions they occupy are manager, assistant manager, cashier, stenographer, teller, and clerk, though not necessarily in that order. The assistant manager is the manager's grandson; the cashier is the stenographer's son-in-law; Mr. Black is a bachelor. Mr. White is twenty-two years old; Miss Ambrose is the tellers step-sister; and Mr. Kelly is the manager's neighbor.

Who holds each postion?

	Manager	Asst. Manager	Cashier	Steno	Teller	Clerk
Black						
White						
Kelly						
Coffee						
Ambrose						
Earnshaw						

SHOPPING

Mrs. Adams, Mrs. Baker, Mrs. Catt, Mrs. Dodge, Mrs. Ennis and that dowdy Mrs. Fish all went shopping one morning at the Emporium. Each woman went directly to the floor carrying the articles which she wanted to buy, and each woman bought only one article. They bought a book, a dress, a handbag, a necktie, a hat, and a lamp.

All the women except Mrs. Adams entered the elevator on the main floor. Two men also entered the elevator. Two women, Mrs. Catt and the one who bought the necktie, got off at the second floor. Dresses were sold on the third floor. The two men got off at the fourth floor. The woman who bought the lamp got off at the fifth floor, leaving that dowdy Mrs. Fish all alone to get off at the sixth floor.

The next day Mrs. Baker, who received the handbag as a surprise gift from one of the women who got off at the second floor, met her husband returning the necktie which one of the other women had given him. If books are sold on the main floor, and Mrs. Ennis was the sixth person to get out of the elevator, what did each of these women buy?

Shopping

	Book	Dress	Handbag	Necktie	Hat	Lamp
Adams	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6
Baker	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6
Catt	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6
Dodge	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6
Ennis	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6
Fish	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6	2 2 3 5 M 6

The Engineer

Smith, Jones and Robinson are the engineer, brakeman and fireman on a train, but not necessarily in that order. Riding the train are three passengers with the same three surnames, to be identified in this problem by a "Mr." before their names.

Mr. Robinson lives in Los Angeles

The brakeman lives in Omaha

Mr. Jones long ago forgot all the algebra he learned in high school.

The passenger whose name is the same as the brakeman's lives in Chicago.

The brakeman and one of the passengers, a distinguished mathematical physicist, attend the same church.

Smith beat the fireman at billiards.

Who is the engineer?

	engineer	breakman	fireman
Smith			
Jones			
Robinson			

	Los Angeles	Omaha	Chicago
Mr. Smith			
Mr. Jones			
Mr. Robinson			

Tea Party

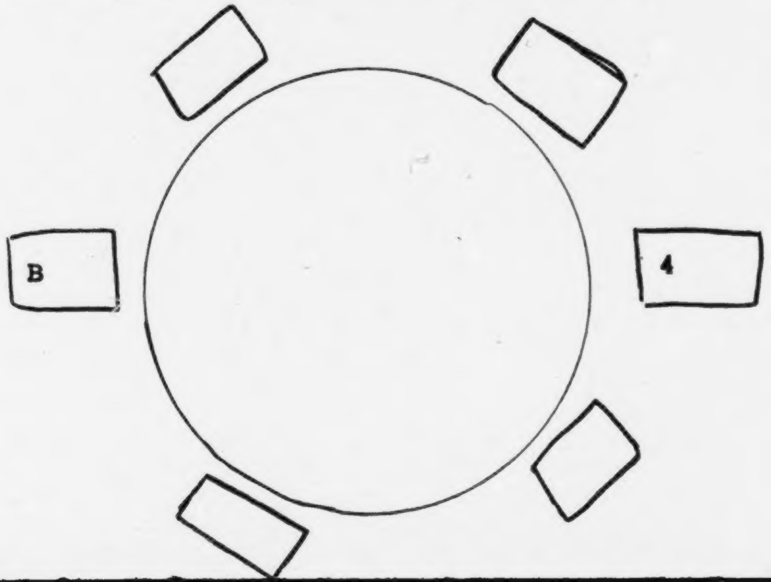
A woman recently gave a tea party to which she invited five guests. The names of the six women who sat down at the circular table were Mrs. Abrams, Mrs. Banjo, Mrs. Clive, Mrs. Dumont, Mrs. Ekwall, and Mrs. Fish. One of them was deaf, one was very talkative, one was terribly fat, one simply hated Mrs. Dumont, one had a vitamin deficiency, and one was the hostess.

The woman who hated Mrs. Dumont sat directly opposite Mrs. Banjo. The deaf woman sat opposite Mrs. Clive, who sat between the woman who had vitamin deficiency and the woman who hated Mrs. Dumont. The fat woman sat opposite Mrs. Abrams, next to the deaf woman and to the left of the women who hated Mrs. Dumont. The woman who had a vitamin deficiency sat between Mrs. Clive and the woman who sat opposite the woman who hated Mrs. Dumont. Mrs. Fish, who was a good friend of everyone, sat next to the fat woman and opposite the hostess.

Can you identify each of these lovely women?

Tea Party

	1 Deaf	2 Talkative	3 Fat	4 Hated Dumont	5 Vitamin Deficiency	6 Hostess
Abrams						
Banjo						
Clive						
Dumont						
Ekwall						
Fish						



Activity #4

Three neighbors, Mr. Carpenter, Mr. Mason, and Mr. Painter, have different occupations. By a strange coincidence, their names are the same as their trades, but not necessarily respectively.

Of the following statements, only one is true.

Mr. Carpenter is not a painter.

Mr. Mason is not a carpenter.

Mr. Carpenter is a carpenter.

Mr. Mason is not a painter.

Who has what occupation?

Acres, Hull, De Maria, and Scott are sports-car drivers whose cars are Mercedes, Austin-Healey, Porsche, and Maseratti--though not necessarily respectively. The following statements are true:

a. Both Hull and the driver of the Maseratti have spent week ends with the driver of the Austin Healey.

b. Acres and De Maria were at the track the day the Porsche driver ran over a Siamese cat.

c. The Maseratti driver who has helped Scott to get membership in the Gear-Box Club is planning to do the same for Acres.

d. Acres had not yet met De Maria.

Question: Who drives which car?

Poker Game

Five men are in a poker game: Brown, Perkins, Turner, Jones, and Reilly. Their brands of cigarettes are Luckies, Camels, Kools, Old Golds, and Chesterfields, but not necessarily in that order. At the beginning of the game, the number of cigarettes possessed by each of the players was 20, 15, 8, 6, and 3, but not necessarily in that order.

During the game, at a certain time when no one was smoking, the following conditions obtained:

a. Perkins asked for three cards.

b. Reilly had smoked half of his original supply, or one less than Turner smoked.

c. The Chesterfield man original had as many more, plus half as many more, plus 2 1/2 more cigarettes than he now has.

d. The man who was drawing to an inside straight could taste only the menthol in his fifth cigarette, the last one he smoked.

e. The man who smokes Luckies had smoked at least two more than anyone else, including Perkins.

f. Brown drew as many aces as he originally had cigarettes.

g. No one had smoked all his cigarettes.

h. The Camel man asks Jones to pass Brown's matches.

How many cigarettes did each man have to begin with, and of what brand?

Andy, Bob, Charlie, Don, and Ed started together for their vacation resorts driving a baby Austin, Buick, Cadillac, Ford, and Oldsmobile. Each agreed to send a postcard to the home of all of the drivers each time he crossed a new state line. The clues are:

a. Don went to a different resort this year to avoid playing golf with the Olds driver.

b. When the group arrived at a crowded ferryboat, Andy, though the last one in line, was the only one to get across on that trip. He received five more cards than he mailed.

c. Don, brother of the Buick driver, dropped back from the group because his daughter was holding hands with the banker's son in the Cadillac ahead.

d. The name of Charlie's car has as many letters as the number of cards he sent out, plus half of that many, plus the fraction 1/2. Andy sent out twice as many cards as Charlie.

e. When the Olds driver had half his number of states, he signaled to Bob that Andy was entering his vacation place.

f. Ed wished the Ford driver luck on the remainder of his journey.

g. Bob sent out the same number of postcards as he received but the Ford driver did not do so well on this score.

Questions: Identify each driver's car.

How many states did each enter?

Post-Test:

1. Joe and the third baseman lived in the same building.
2. Bob, Joe and Frank and the catcher were beaten at golf by the second baseman.
3. Ed was a very close friend of the catcher.
4. The center fielder was taller than the right fielder.
5. The shortstop, the third baseman and Frank each liked to go to the races.
6. The pitcher's wife was the third baseman's sister.
7. Bill's sister was engaged to the second baseman.
8. Bob and Harry each won $5 from the pitcher at poker.
9. The catcher and the third baseman each had two children.
10. Jim decided to get a divorce.
11. All of the battery and infield, except Harry, Joe, and Ed were shorter than Jim.
12. Bill and the outfielders like to play Gin Rummy together whenever they could.
13. Jack was taller than Frank. Tom was shorter than Frank. Each weighed more than the 1st baseman.
14. One of the outfielders was either Tom or Ed.
15. Bill, Bob and Jack, the center fielder and the right fielder were bachelors. The others were married.
16. Bob, Ed and the shortstop were teetotalers.

What is the name for the man at each position?

Post-Test:

In a certain mythical community, politicians always lie, and non-politicians always tell the truth. A stranger meets three natives, and asks the first of them if he is a politician. The first native answers the question. The second native then reports that the first native denied being a politician. Then the third native asserts that the first native is really a politician.

How many of these natives are politicians?

On a certain train, the crew consists of three men, the brakeman, the fireman, and the engineer. Their names listed alphabetically are Jones, Robinson, and Smith. On the train are also three passengers with corresponding names, Mr. Jones, Mr. Robinson, and Mr. Smith. The following facts are known:

a. Mr. Robinson lives in Detroit.
b. The brakeman lives halfway between Detroit and Chicago.
c. Mr. Jones earns exactly $20,000 a year.
d. Smith once beat the fireman at billiards.
e. The brakeman's next-door neighbor, one of the three passengers mentioned, earns exactly three times as much as the brakeman.
f. The passenger living in Chicago has the same name as the brakeman.

What was the engineer's name?

CLUSTER: MANUFACTURING

AREA: DESIGN

TITLE: THREADS AND ANGLES

BY: JIM MADEHEIM
SAHUARITA HIGH SCHOOL
MATH DEPARTMENT

Rationale:

This unit will demonstrate the use of trig as used in the machine trades. It is assumed that you are familiar with the use of the sine, cosine, and tangent functions.

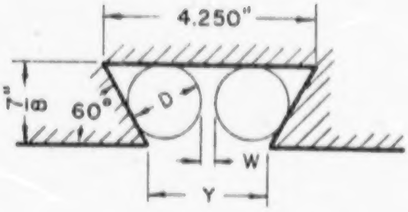
The problems use the language of these trades, so that you will become familiar with words such as pitch, dovetail, Acme threads, etc.

The new ideas and words are not difficult to learn and your practice in trig won't be hurt if you do not learn them. Activities 4,5, and 6 are optional, because they cover areas that require a deeper knowledge of the machine trades.

Behavioral Objectives:

Given problems applicable to the machine trades that involve the use of trig to find their solution, you will be able to solve the problems with four digit accuracy at least 75% of the time.

Pre-evaluation:



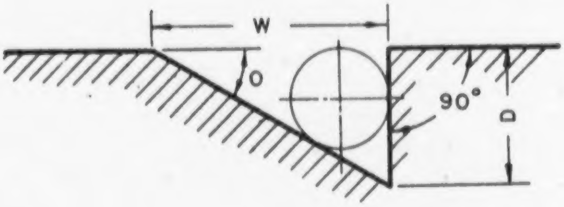
1. Find W using the above illustration and using 3/4" plugs. ____________

2. Find W as above using 9/16" plugs. ____________

3. Using the picture below, find the size of the plug to be usedif:

D = 2" O = 30°

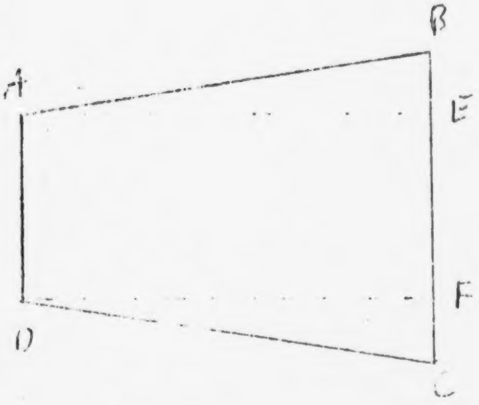
D = 1 1/2" O = 25°



Information Sources:

1. Read Data Brief # 1 "Taper."
2. Read Data Brief # 2 "Helix."
3. Read Data Brief # 3 "Measuring dovetails."
4. Read Data Brief # 4 "Measuring pitch diameter."
5. Read Data Brief # 5 "Best Wire Size."
6. Read Data Brief # 6 "Measuring a cut accurately."
7. Read Data Brief # 7 "Practical problems."

Data Brief # 1



In a taper as pictured above, B

$$BE + EF + FC = BC$$

$$EF = AD$$

$$BE = FC$$

$$2FC + AD = BC$$

$$2FC = BC - AD$$

$$BE = FC = (BC - AD)/2$$

Data Brief # 2

Helix: The curved line which the point of a tool makes on the surface of a cylinder in cutting a thread, is a helix, commonly called a spiral.

Fig. 1. The horizontal distance advanced by the helix in making a complete revolution is called the lead of the screw. If one turn of the helix could be unwrapped from the surface of the cylinder it would borm the hypotenuse AC , of the right triangle ABC, in which the side AB is equal to the lead and the side BC is equal to the circumference. The angle BCA is the helix angle.

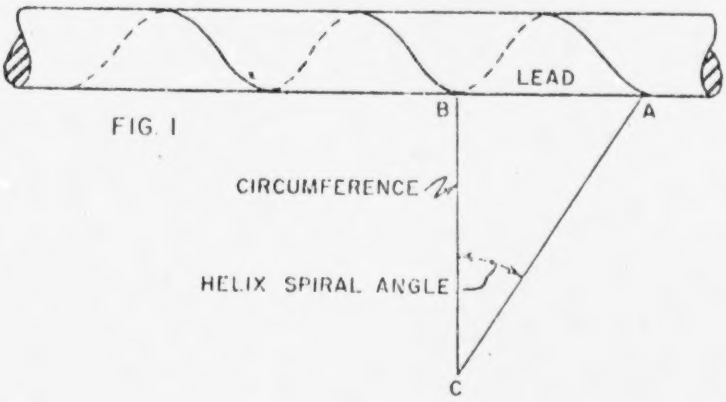


FIG. I

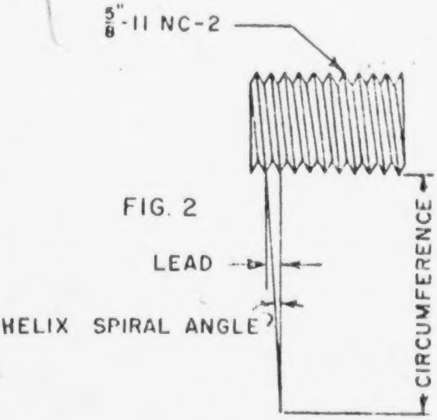


FIG. 2

In cutting the screw threads on a lathe, it is necessary to rotate the work and feed the tool parallel to the center line. The tool as it cuts will describe a helix curve on the work. This curve is often referred to in the shop as a "spiral." The amount of feed per revolution of the work determines the lead of the screw thread cut, as by expressed by the tangent ratio.

$$\text{tangent} = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\begin{array}{c}\text{tangent of helix}\\ \text{(spiral) angle}\end{array} = \frac{\text{lead}}{\text{circumference}}$$

This gives us a method for figuring the helix(spiral) angle which it is necessary to know in order to grind the proper clearance on the threading tool and in setting the cutter on the thread milling machine.

Helix angles for drills, cutters and gears

The helix for these tools is similar to the helix for screw threads. The horizontal distance advanced by the helix in making a complete revolution is called the lead, Fig. 4. The side BC of the triangle ABC is equal to the circumference and the hypotenuse AC is the helix. The angle BAC is the helix angle.

Rule. The tangent of the helix angle is equal to the circumference divided by the lead.

tangent of a helix

$$\text{tangent of a helix angle} = \frac{\text{circumference}}{\text{lead}}$$

FIG. 3

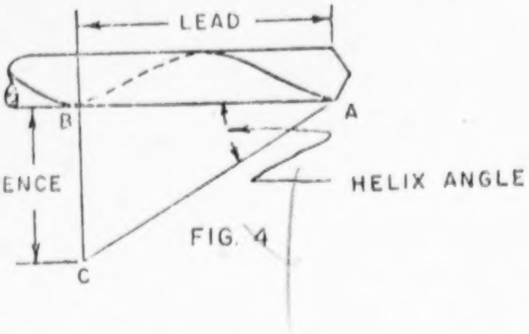


FIG. 4

Compare the angle shown as the helix(spiral) angle in Fig. 1, with the location of the helix(spiral) angle in Fig. 4, on this page. Notice that it is not the same angle in each case. In order to understand this difference, suppose a triangle with the same shape as that shown in Fig. 5 was cut out of paper and wrapped around a rod (Fig. 6), then the helix (spiral) curve shown in Fig. 7 would result. If this curve represents a helix(spiral) of a screw thread, the angle will be 70°. If this curve represents the helix(spiral) of a twist drill, a spiral gear, or a cutter, then the angle will be 20°.

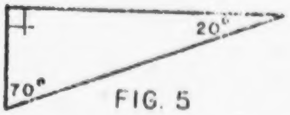


FIG. 5

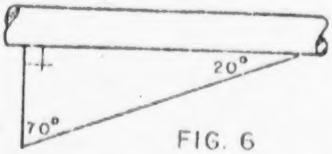


FIG. 6

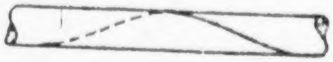


FIG. 7

Data Brief # 3

A common way to measure dovetails accurately is to "mike" over a pair of plugs, as shown in Fig. 1, for the distance X. (For this example, if your given the 3.1416 measurement as being accurate, then you are proving that the 60° is accurate,if X comes out right.) In order to have this distance X meaningful, it is necessary to first find what X should be theoretically. To do this, first find Y. From Fig. 2 below, Y is equal to the radius of the plug plus the distance AC, which is a side of a right triangle. Thus X should equal 3.1416 + 2Y.

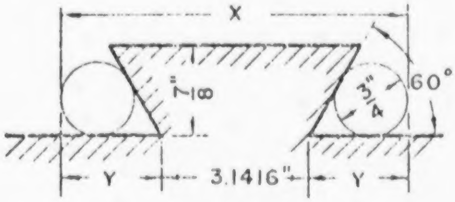


FIG. 1

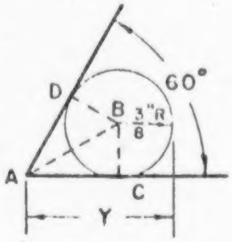


FIG. 2

Data Brief # 4

This "three wire" method of measuring the pitch diameter of screws is recommended by the Bureau of Standards as the best means of securing uniformity. The best size wire is that which touches the thread at the pitch line (the middle of the sloping sides). That is, the sides of the thread are tangent to the wire in two places, and these points of tangency are on the pitch line. (See pictures on the next page).

Three wire method.

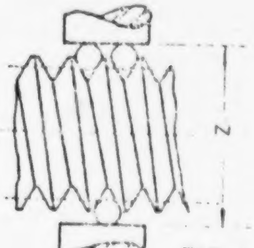


FIG 1

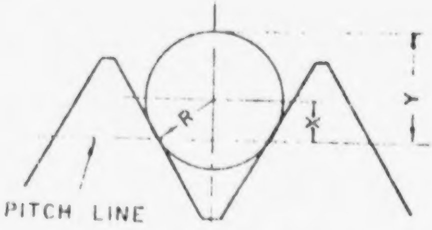
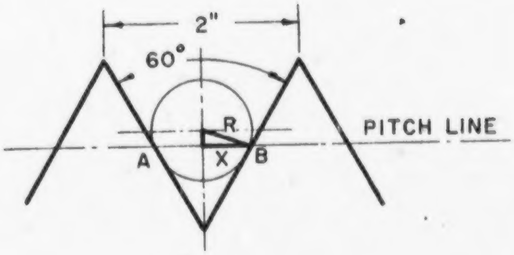


FIG 2

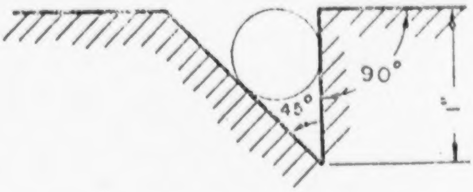
Data Brief # 5

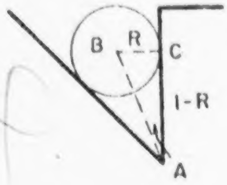


The best wire for measuring threads is a wire that touches the side of the thread at the pitch line. The size of this wire may be found as described by the method on the following page. (Note that the dimensions are different than those on this page.)

Data Brief # 6

To accurately measure a cut, as shown, the size of a plug which will be flush with the top surface is calculated. The plug is then turned to size and used to test the cut.

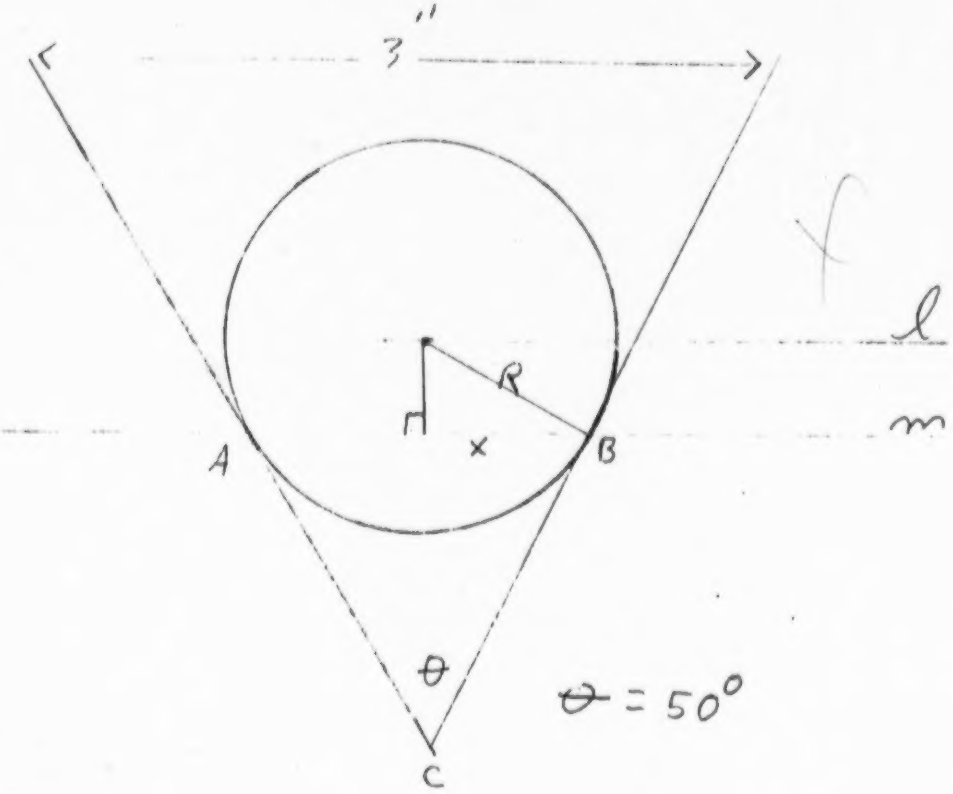




In the above drawing, you would have to calculate the distance BC which is the radius (R) of this circular cross section of the plug.

Data Brief # 7

The problems in activity # 7 are made up of all the ideas presented in this package plus any other idea having to do with trig that is appropriate.



line m is drawn thru both points of tangency.

line l is drawn thru the center of the circle parallel to m

line m is the pitch line

angle CAB = angle CBA.   Why?

angle B in the triangle is 40°. Why?

X = 1.5"   Why?

R = 1.5(cos 40°)   Why

Once you have found the radius of the wire (R), then you can find the diameter of the wire that is required.

The following material has been deleted: Activity #1-7.

Post-evaluation:

1. The teacher will pick three problems for you to do from the activity sheets you have just completed. Some of the dimensions will be changed.

2. Take the pre-test.

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RATIONALE

[illegible] this package in order to find out why you should learn about tape production.

Find out about some of the career and entertainment and hobby values that knowledge about taping techniques can give you.

NUMBER: 1

CLUSTER: CONSTRUCTION

AREA: CIVIL ENGINEERING

TITLE: 3 + 4 = 5

Rationale:

Any two or more forces acting on a body (in a plane) can be replaced by one force that will produce the same result. This idea is thoroughly explored in this package.

Behavioral Objective:

Given a problem that asks for the resultant of a set of forces (in a plane), you will be able to find this resultant using the parallelogram and vector methods, both, with 60% accuracy. (You will also be given a grade for specific homework problems that are asked for. All tests are open book).

Pre-test:

Pages 33 - 35 in the text. Problems 2.31, 2.41, 2.46.

Information Sources:

Text book:	"Vector Mechanics for Engineers"
Data Brief # 1	"Parallelogram law"
Data Brief # 2	"i and j"
Data Brief # 3	"Eqilibrium"

Data Brief # 1

THE FOLLOWING LAW IS BASED ON EXPERIMENTAL EVIDENCE: IT CANNOT BE PROVED OR DERIVED MATHEMATICALLY.

Experimental evidence shows that two forces P and Q acting on a particle A (Fig. 1) may be replaced by a single force R which has the same effect on the particle (Fig. 3). This force is called the resultant of the forces P and Q and may be obtained, as shown in Fig 2, by constructing a parallelogram, using P and Q as two sides of the parallelogram.

The diagonal that passes thru A represents the resultant. This is known as the parallelogram law for the addition of two forces.

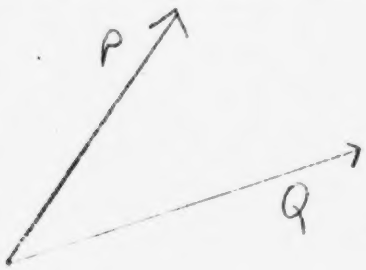


Fig 1

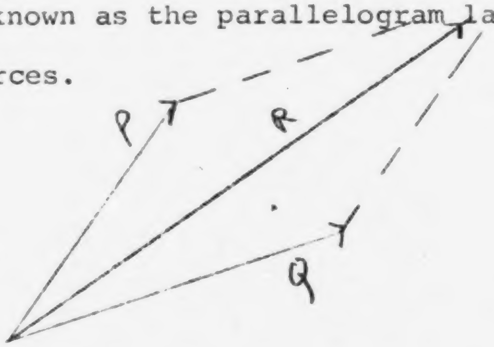


Fig. 2

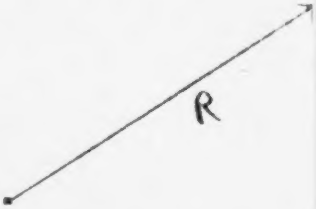
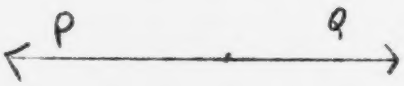
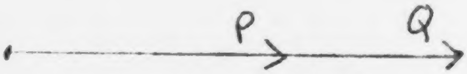


Fig. 3

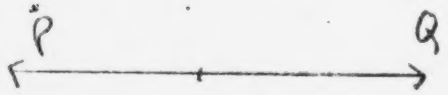
If P = 25 lbs, and Q = 15 lbs, then R = 15 lbs (in the P direction). Fig. 4.



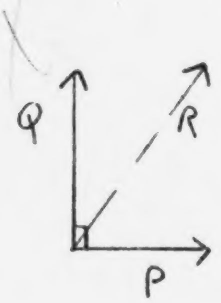
If P = 20 lbs. and Q = 35 lbs, then R = 55 lbs. in the same direction as both P and Q are pointing. Fig. 5



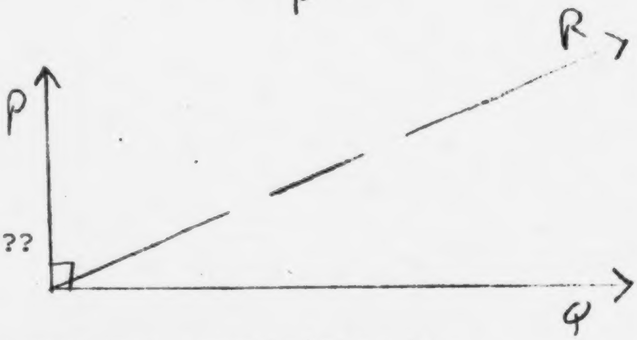
IF P = 30 lbs. and Q = 30 lbs as in Fig. 6, then R = 0.



In Fig 7, if P = 3 lbs.
Q = 4 lbs.
then R = 5 lbs. WHY????



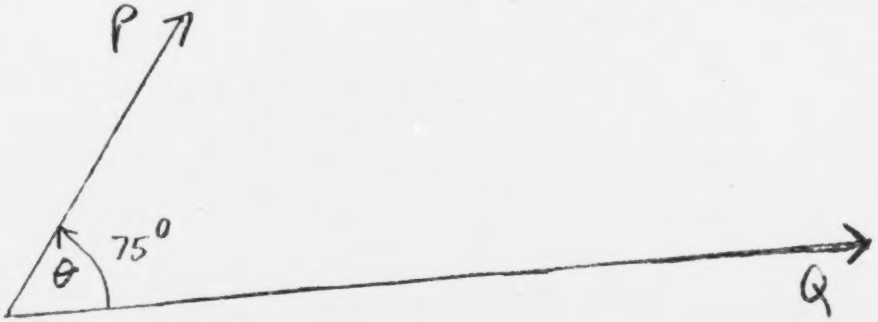
In Fig 8, if P = 5 lbs.
Q = 12 lbs.
then R = 13 lbs. WHY?????

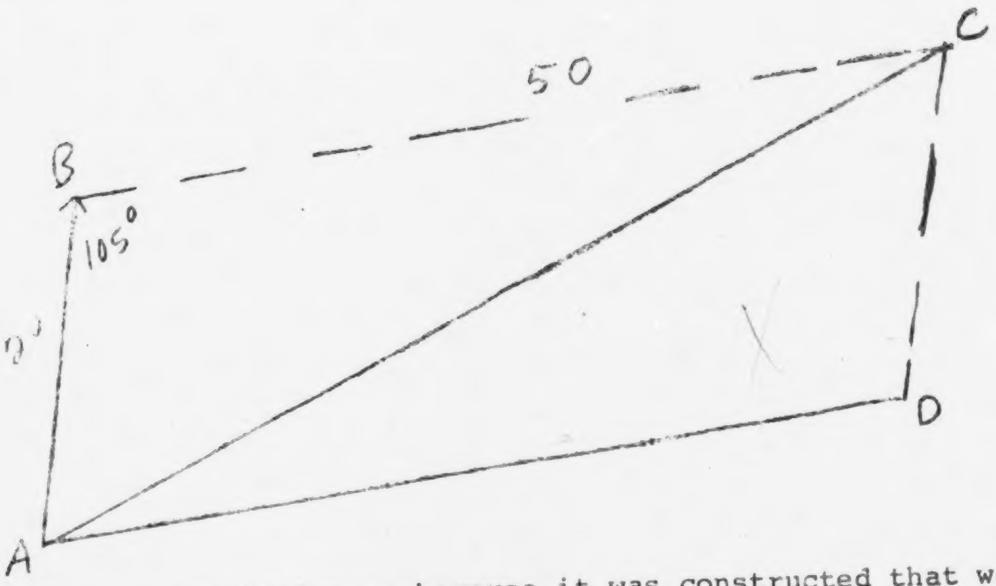


Vectors are defined as mathematical expressions possessing magnitude and direction, which add according to the parallelogram law. All vector quantities will be represented by the oversized capitol letters (P,Q,R,Z)

The above cases are special and the resultant is easily calculated. The next case is a bit harder.

P = 20 lbs.
Q = 50 lbs.
θ = 75°
Find R





ABCD is a parallelogram because it was constructed that way.

ABCD Therefore BC = AD

AD is a line whose length represents the force of 50 lbs.

Thus, BC = 50. AB = 20.

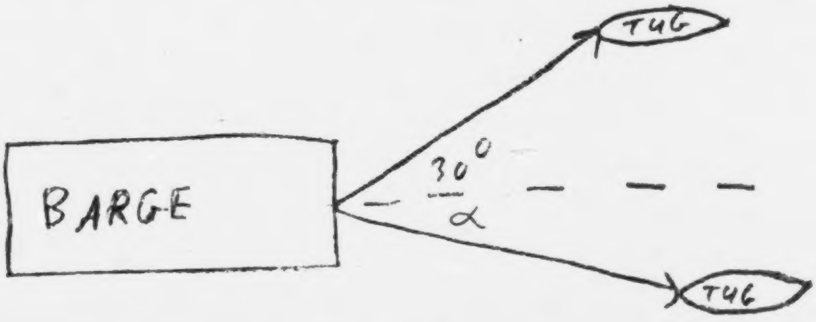
ANgle ABC = 180° - θ, = 105°

How do you find the length of the third side of a triangle knowing two sides and their included angle?

If you want to find angle BAC, you can use the law of sines.

Example.

A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is a 5000 lb force directed along the axis of the barge, find the value of α such that the tension in rope 2 is a minimum.



YOU MUST BECOME FAMILIAR WITH THE VECTOR QUANTITIES i AND j. START USING THEM NOW!!!!!!!

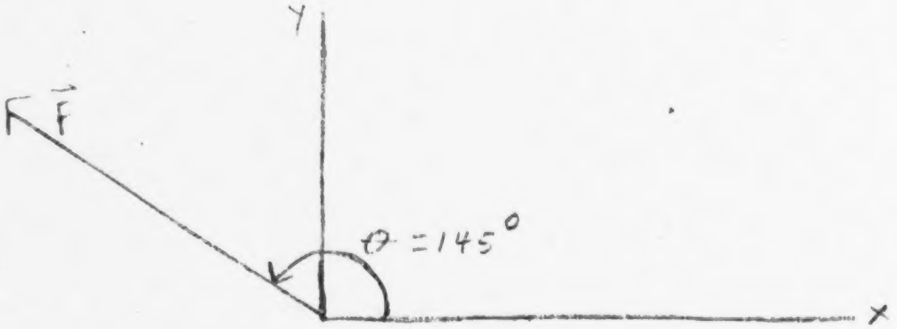
example

An 800 lb force is exerted on bolt A as shown below. Determine the horizontal and vertical components of the force.

$F_x = F_x i = F\cos\theta i = 800(\cos 145°)i = -800(\cos 35°)i = -655i$

$F_y = F_y j = F\sin\theta j = 800(\sin 145°) = +459j$

(1) $F_x = -655i$ $F_y = +459j$ or $F = -655i + 459j$



On the next page is another method for obtaining the same result.

The i vector always points toward the + x direction and the j vector always points toward the + y direction. The scalar component F_x is positive when the vector F_x has the same sende as the unit vector i.

Otherwise F_x is negative.

Activity # 2

2.1 In removing a nail, a force of 250 lb is applied by a hammer in the direction shown. What are the horizontal and vertical components of this force?

2.2 Determine the components of the 300 lb force in directions parallel and perpendicular to beam AB.

2.3 The force P must have a 60 lb component acting up the incline. Determine the magnitude of P and of its component perpendicular to the incline.

2.4 Determine the x and y components of each of the forces shown.

2.5 The tension in the telephone pole guy wire is 390 lb. Determine the horizontal and vertical components of the force acting on the anchor A.

2.6 Determine the x and y components of the forces shown.

2.7 The x and y components of a force F are shown. Determine the magnitude and direction of the force F.

2.8 Determine the resultant of the four forces shown.

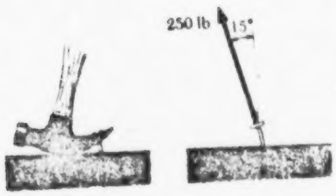


Fig. 2.1

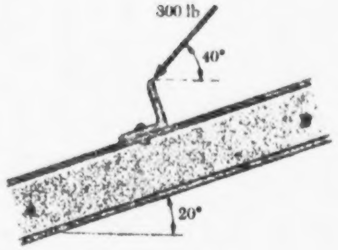


Fig 2.2

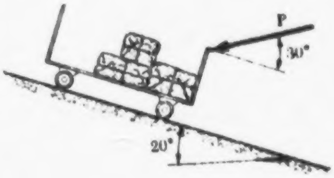


Fig. 2.3

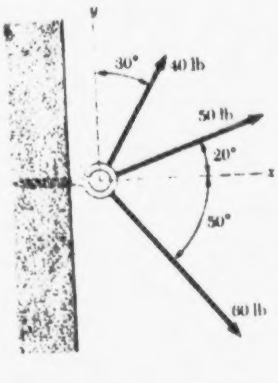


Fig. 2.4

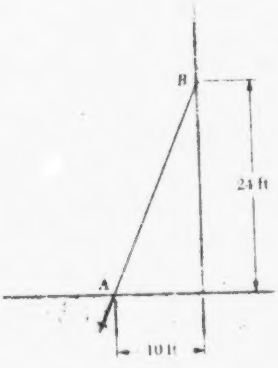


Fig. 2.5

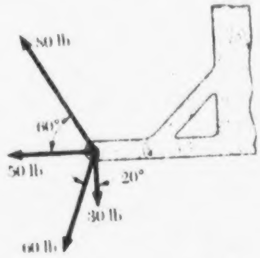


Fig 2.8

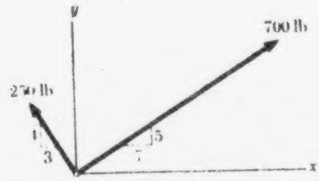


Fig. 2.6

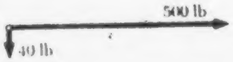


Fig. 2.7

2.9 A hoist trolley is subjected to the three forces shown. The direction of the force F may be varied. If possible, determine the direction of the force F so that the resultant of the three forces is vertical, knowing that the magnitude of is (a) 240 lb, (b) 140 lb.

2.10 Two cables which have know tensions are attached at point B. A third cable AB is used as a guy wire and is also attached at B. Determine the required tension in AB so that the resultant of the forces exerted by the three cables will be vertical.

2.11 The resultant of the three forces shown must be a 100 lb force directed to the right along line a-a. Determine the required magnitude and direction of the force F.

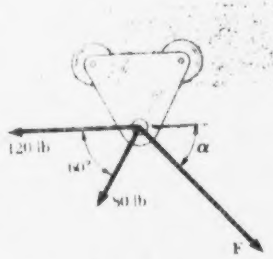


Fig. 2.9

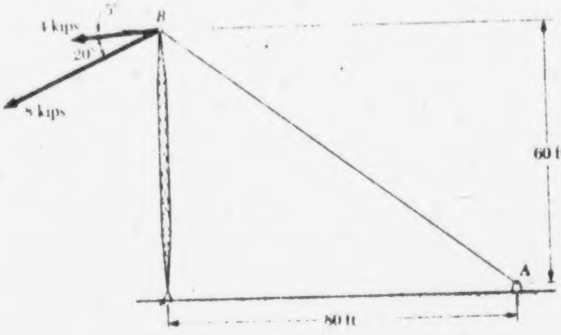


Fig. 2.10

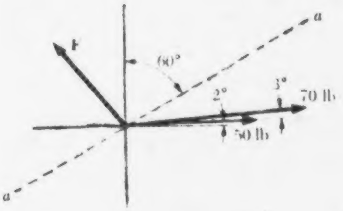


Fig. 2.11

Activity # 3

3.1, 3.2 Two cables are tied together at C and loaded as shown. Determine the tension in AC and BC.

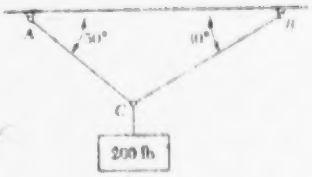


Fig 3.1

3.3 If the length of cable BC is increased to 10 ft in problem 3.2, determine the new tension in AC & BC.

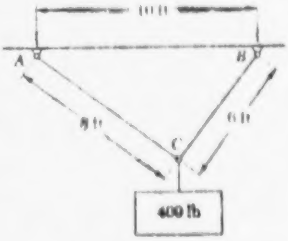


Fig 3.2

3.4 A 600 lb block is supproted by the two cables AC & BC. (a) For what value of α is the tension in AC minimum? (b) What are the corresponding values of the tension in cables AC & BC? (c) the value for α for which the larger on the cable tensions is as small as possible, (d) the corresponding values of the tension in cables AC and BC. (Use the table below). Use the formulas below.

α	T_{AC}	T_{BC}
5°		
20°		
30°		
40°		
50°		
60°		
70°		
85°		

$$T_{BC} = \frac{1200}{\tan\alpha + \sqrt{3}}$$

$$T_{AC} = \frac{T_{BC}}{2\cos\alpha}$$

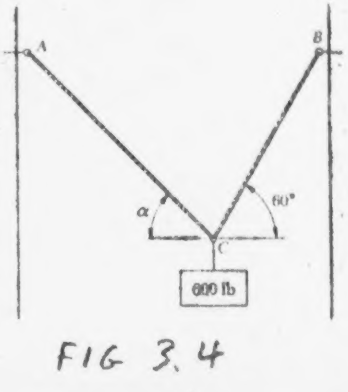


FIG 3.4

3.5 A man stretches an elastic cord AC by applying his finger at B. Determine the magnitude and direction of the force exerted by the man, knowing that the tension in both parts of the cord is 5 lb.

3.6 Two ropes are tied together at C. If the maximum permissible tension in each rope is 750 lb, what is the maximum force that may be applied? In what direction must this force act?

3.7 Two forces P and Q are applied to the aircraft connection shown. At a certain instant, when the connection is in equilibrium, it is found that T_1 = 560 lb and T_2 = 120 lb. Determine the corresponding values of P and Q .

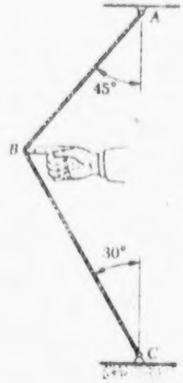


Fig 3.5

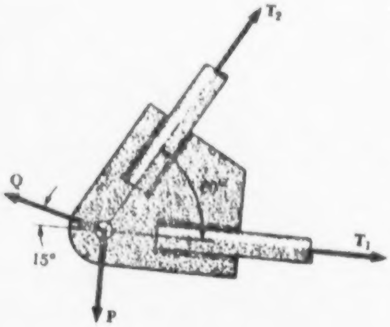


Fig 3.7

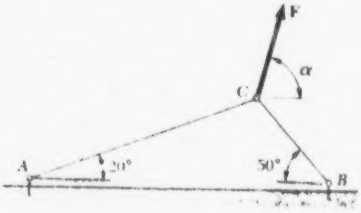


Fig 3.6

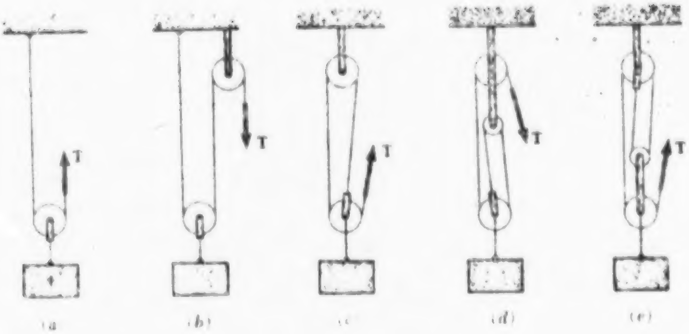


FIG 3.8

3.8 A 600 lb crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. (The tension in the rope is the same on each side of a simple pulley.)

3.9 Solve parts b and d of prob. 3.8 assuming that the free end of the rope is a-tached to the crate.

3.10 A 1500 lb crate is lifted by a crane cable CD. A cable sling ACB is 5 ft long and can be attached to the crate in each of the two ways shown. Determine the tension in the cable sling in each case.

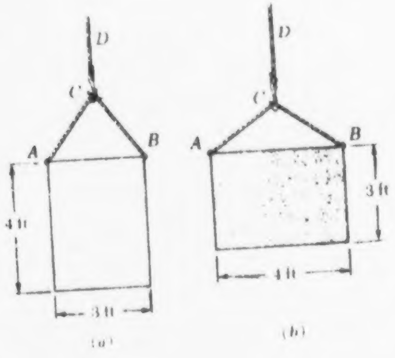


FIG 3.10

3.11 A portable bin and its contents weigh 750 lb. Determine the shortest sling (chain) ACB which may be used to lift the loaded bin if the tension in the chain sling is not to exceed 900 lb.

First answer this question: What is the heaviest load that can be lifted by a chain that is used as in the picture, if the chain is of length 500 ft?

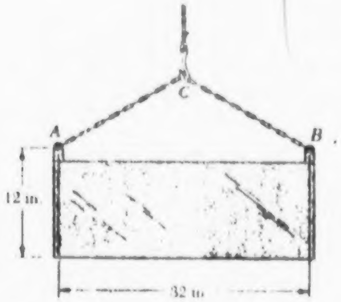
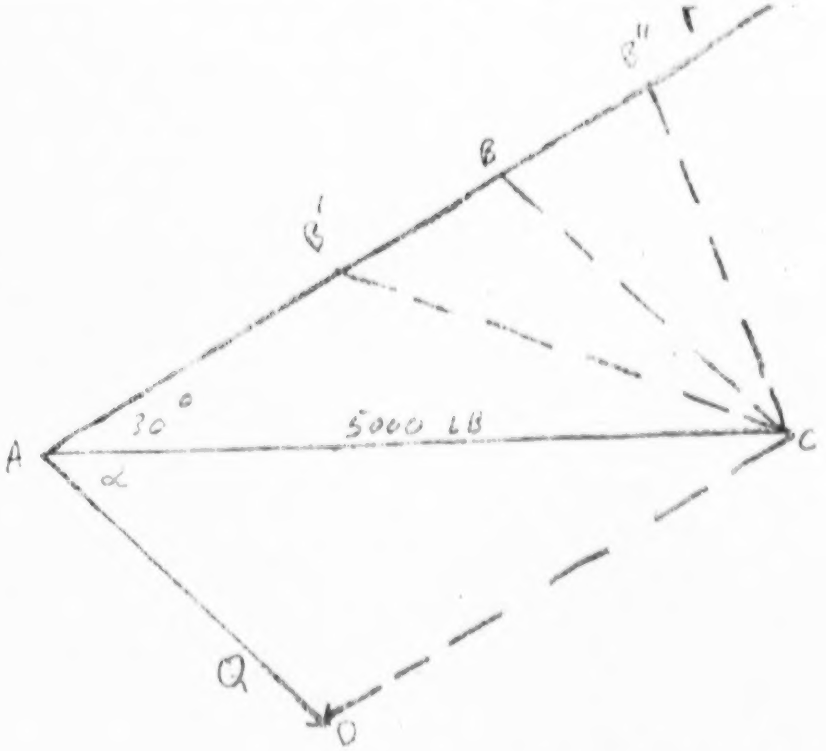


FIG. 3.11



In this problem, we want the length of line AD to be a minimum. The length of AC is fixed. The length of AB is not fixed, but its direction is. Since this is to be a parallelogram, line AD and BC will be equal.

Thus when you find the mimimum of line BC, you have found the minimum of line AD.

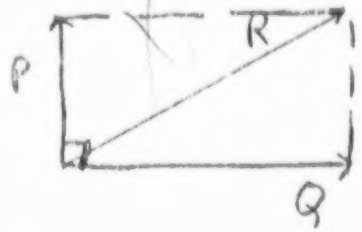
Using point C as a pivot point (why?) swing line BC until it looks shortest. Thus B'C is longer than BC, and B"C is longer than BC, and BC is not the shortest distance from C to line AB.

You should now be able to guess that $\alpha = 60°$.

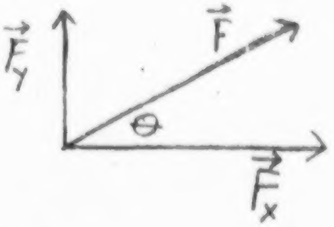
Data Brief # 2

We shall at this point, introduce two vectors of magnitude 1 (one), directed respectively along the x and y axes. These vectors are called unit vectors and are denoted by i and j, respectively.

Referring to the picture at the right, $R = P + Q$ (Vector addition!!)



The picture to the right, here, is the same thing with only the new notation addedd.



$$F = F_x + F_y$$

$$F_x = F_x i$$

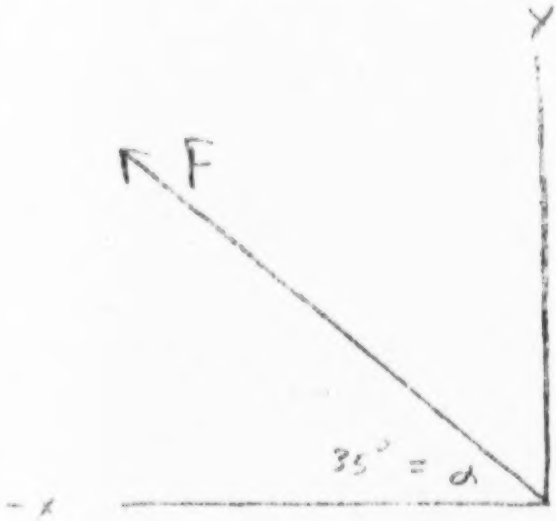
$$F_y = F_y j$$

$$F_x = F\cos\theta$$

$$F_y = F\sin\theta$$

θ is the angle between F and the x axis, measured ccw from the positive x axis

F is not a vector. It is the magnitude of the vector F.



$$\vec{F}_x = -F_y i = -F\cos\alpha\, i = -800\cos 35°\, i = -655 i$$

$$\vec{F}_y = F_y j = +459 j$$

When three or more forces are to be added, no practical trig solution may be optained from a series of parallelograms. In this case an analytic solution may be obtained by resolving each force into two rectangular components. Consider the pictures below.

$$R = P + Q + S$$

$$R_x i + R_y j = P_x i + P_y j + Q_x i + Q_y j + S_x i + S_y j$$

$$= (P_x + Q_x + S_x)i + (P_y + Q_y + S_y)j$$

This means that

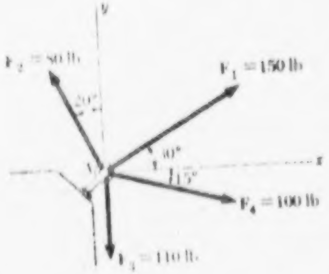
$$R_x = P_x + Q_x + S_x \qquad R_y = P_y + Q_y + S_y$$

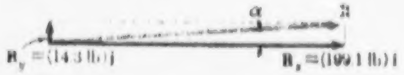
$$R = R_x i + R_y j$$

Example: Four forces act on bolt A as shown. Determine the resultant of the forces on the bolt.

$R = R_x i + R_y j$

Force	Magnitude	x component	y component
F_1	150	$F_1 \cos 30° = 129.9$	$F_1 \sin 30° = +75$
F_2	80	$-F_2 \cos 70° = -27.4$	$F_2 \sin 70° = +75.2$
F_3	110	$F_3 \cos 90° = 0.0$	$-F_3 \sin 90° = -110$
F_4	100	$F_4 \cos 15° = 96.6$	$-F_4 \sin 15° = -25.9$
		$R_x = +199.1$	$R_y = +14.3$





$R = (199.1 \text{ lb})i + (14.3 \text{ lb})j = 199.14$ lbs $\measuredangle$ 4.1°

Data Brief # 3

In the preceeding sections, we discussed the methods for determining the resultant of several forces acting on a particle. Although this has not occurred in any of the problems considered so far, it is quite possible for the resultant to be zero. In such a case, the net effect of the given forces is zero, and the particle is said to be in equilibrium.

Example: A small motor boat is moored by means of three ropes tied to posts on the banks of a stream. The stream flow exerts a force on a force on the boat which acts directly downstream. The tensions in ropes A and B are measured and found to be A = 120 lb and B = 80 lb. Determine the magnitude of the force exerted by the flow and the tension in rope C.

The boat is taken as a free body. It is acted upon by four forces directed as shown. Each force is resolved into its x and y components.

$$A = -(120\text{lb})j$$

$$B = -(80\text{lb})\cos 30^\circ i + (80\text{ lb})\sin 30^\circ j$$

$$C = C\sin 20^\circ i + C\cos 20^\circ j$$

$$D = Di$$

$$B = -(69.3 \text{ lb})i + (40 \text{ lb})j$$

$$C = 0.342Ci + 0.940Cj$$

$$R = A + B + C + D = 0$$

thus

$$-(120j - 69.3i + 40j + .342Ci + ,940Cj + Di = 0$$

Factoring;

$$(-69.3 + .342C + D)i + (-120 + 40 + .940C)j = 0$$

This equation will be satisfied, if and only if, the coefficients of i and j are equal to zero. Therefore -

$$-69.3 + .342C + D = 0$$

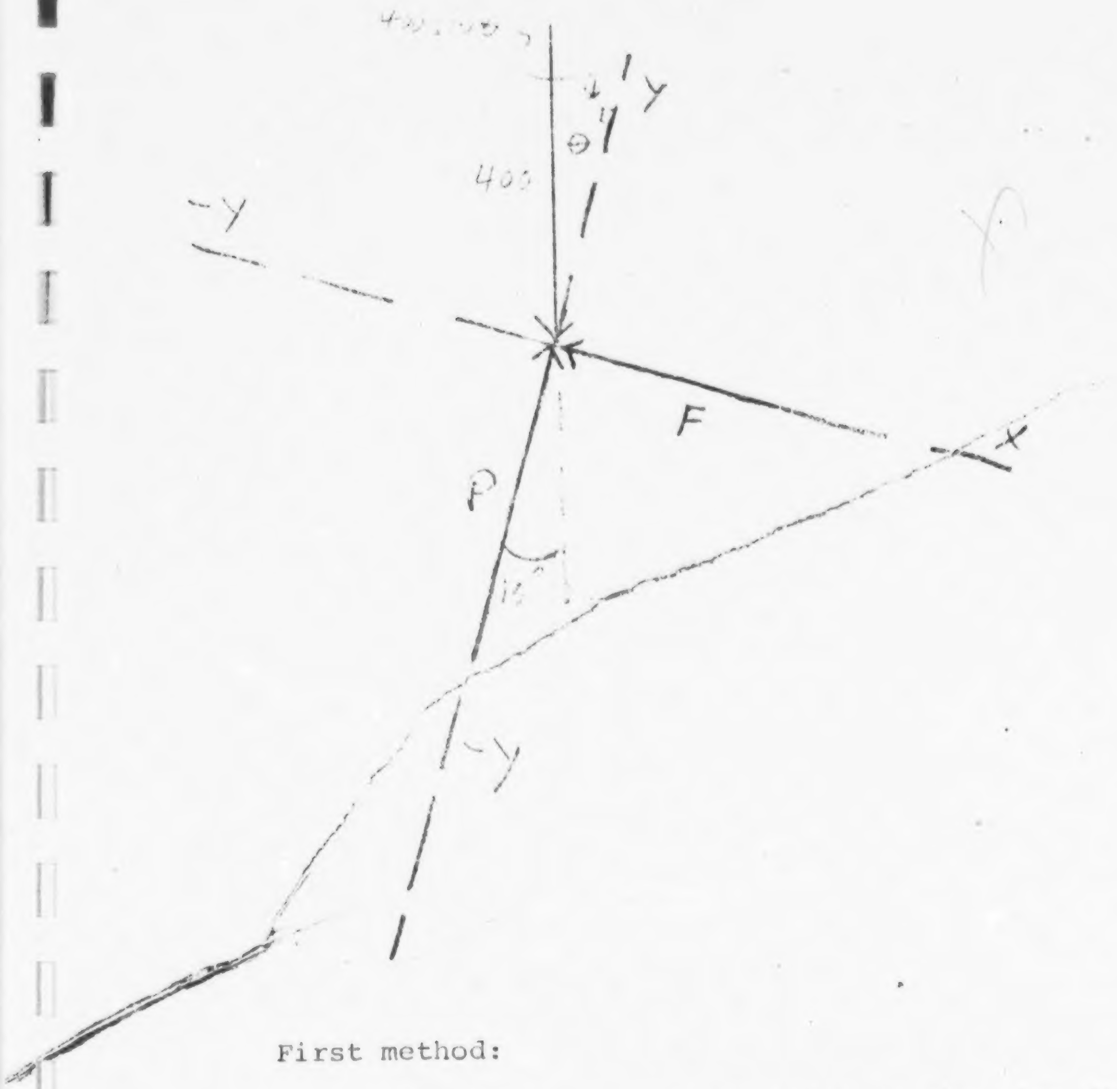
$$-120 + 40 + .940C = 0$$

$$C = +85.1 \text{ lb}$$

$$D = +40.2 \text{ lb}$$

Determine the magnitude and direction of the smallest force which will maintain the package shown in equilibrium. Note that the force exerted on the package by the rollers is perpendicular to the incline.

?
PICTURE



First method:

We choose the y-axis to be in the same direction as P.

Thus $\theta = 15°$

The force F keeps the package from rolling down and so it must be parallel to the rollers and push directly back up the incline to be a minimum.

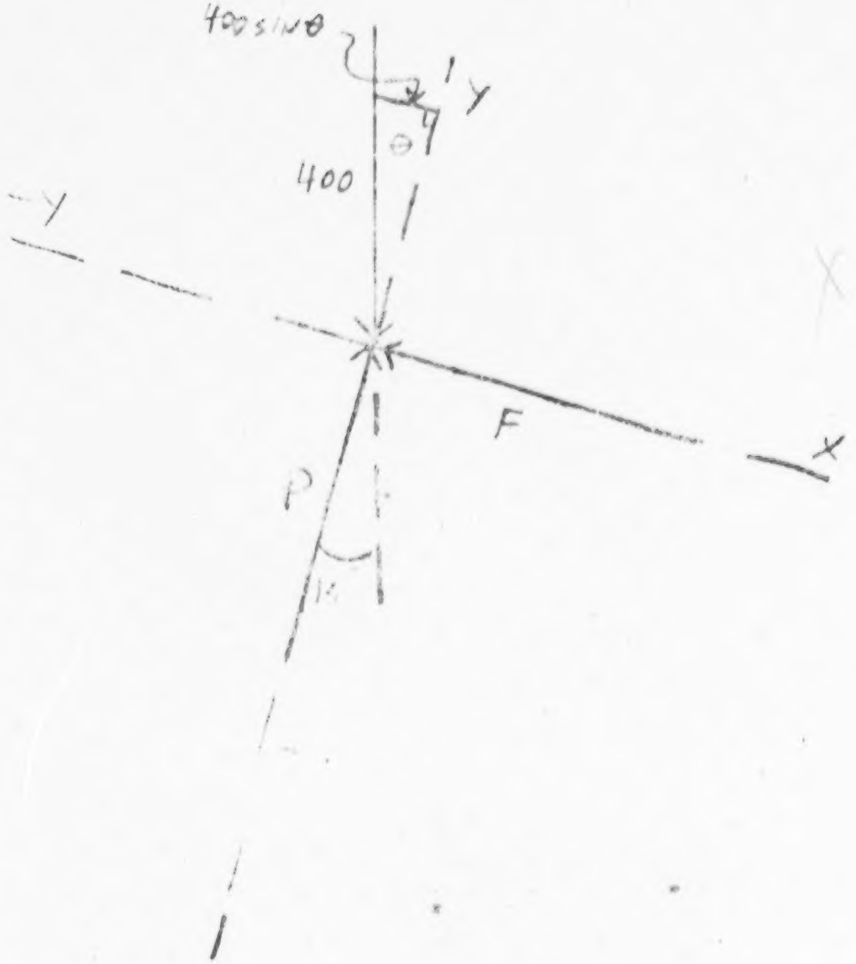
Thus the x-axis is in the same direction as F.

Now each force is broken down into its x and y components.

$< 0 = F_x = (400 \sin\theta) i - (F) i$

$F = 400(\sin 15°) = 103.6$ lb

$F = 103.6$ lb $^{F}\triangle$ 15°



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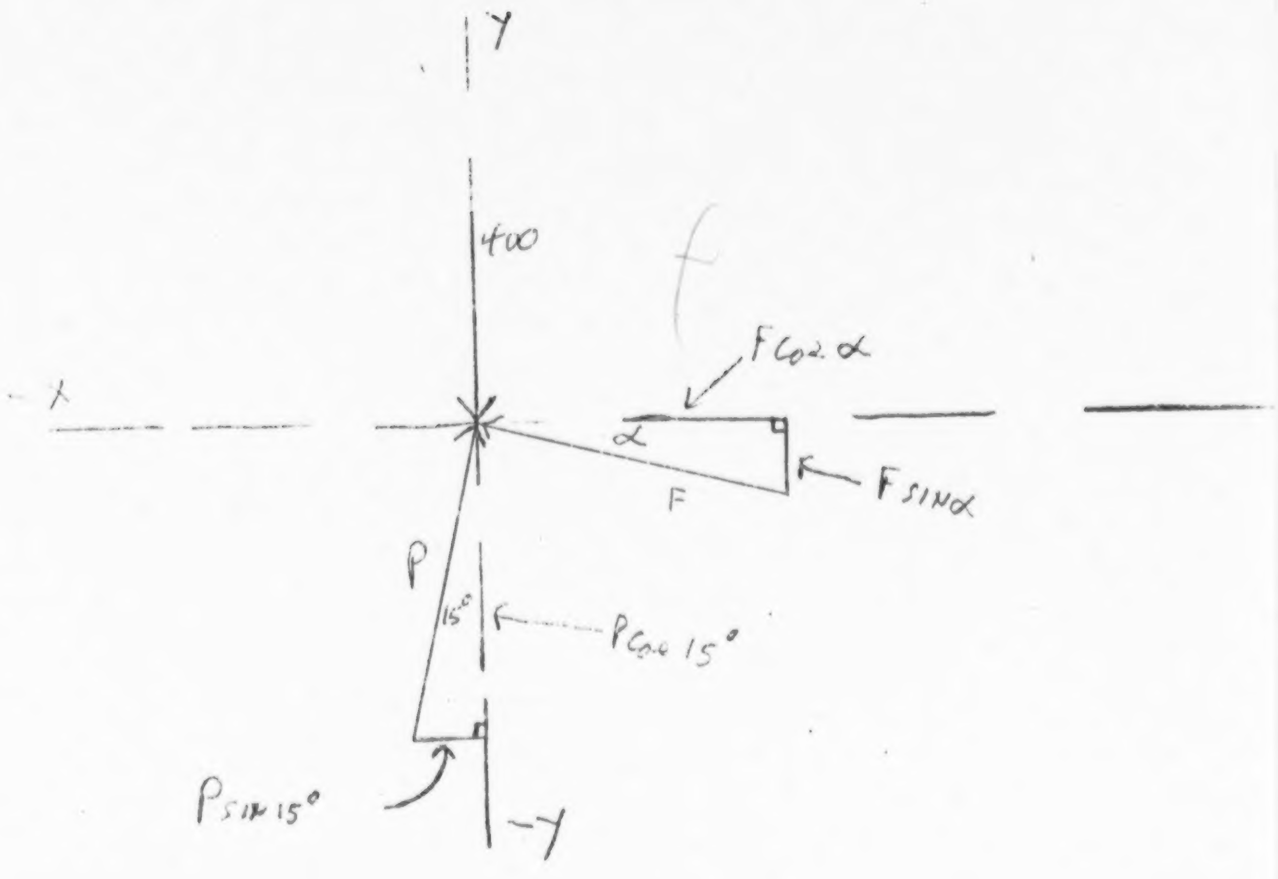
$F = 400(\sin 15°) = 103.6\text{ lb}$

$F = 103.6\text{ lb} \angle 15°$



$0 = F_x = (-F\cos\alpha)i + (P\sin 15°)i$

$0 = F_y = (-400)j + (P\cos 15°)j + (F\sin\alpha)j$

$0 = -400 + P\cos 15° + F\sin 15°$ (1)

$0 = -F\cos 15° + P\sin 15°$ (2)

$P = F\ (F\cos 15°)/(\sin 15°)$ (3) from (2)

$$0 = -400 + \frac{F\cos 15°}{\sin 15°}(\cos 15°) + F\sin 15° \quad \left[\text{FROM (1)+(3)}\right]$$

$$400 = \frac{F\cos^2 15° + F\sin^2 15°}{\sin 15°}$$

$$400 \sin 15° = F(\sin^2 15° + \cos^2 15°) = F = 103.6 \text{ lb}$$

Activity # 1

1.1 Determine graphically the magnitude and direction of the resultant of the two forces shown.

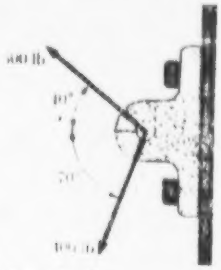


Fig 1.1

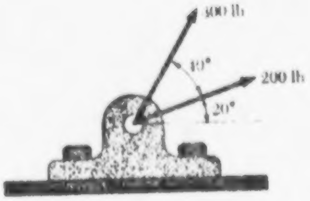


Fig 1.2

1.2 Follow the directions for 1.1.

1.3 Two structural members B and C are riveted to the bracket A. Knowing that the tension in member B is 2500 lb and that the tension in C is 2000 lb, determine graphically the magnitude and direction of the resultant force acting on the bracket

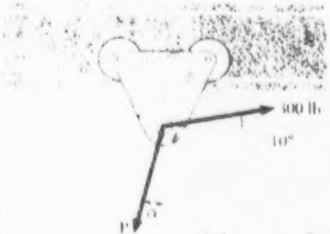


Fig. 1.5

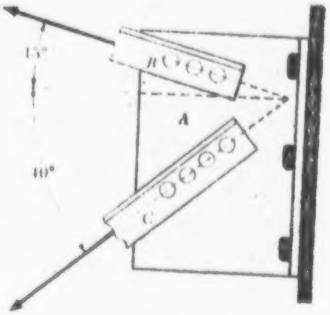


Fig 1.3

1.4 Solve 1.3 by trig.

1.5 Determine by trig the magnitude and direction of the force P so that the resultant of P and the 300 lb force is a vertical force of 900 lb directed downward.

1.6 A disabled car is being pulled by means of two ropes. Knowing that the tension in rope 1 is 150 lb, determine the magnitude and direction of the tension in rope 2 so that the resultant is a 200 lb force parallel to the axis of the car.

1.7 If the angle $\alpha = 30°$, determine the tension T_2 so that the resultant force exerted at A is parallel to the axis of the automobile.

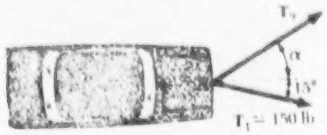


Fig 1.6, 1.7

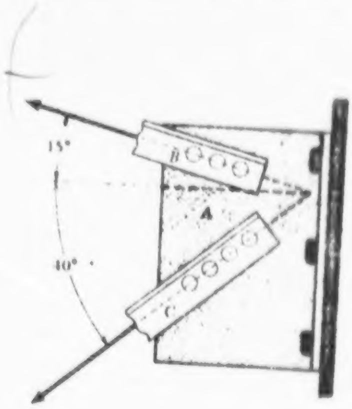


Fig 1.8

1.8 The tension in member B is 1500 lb. If the resultant of the forces exerted by members B and C is to be horizontal, determine the tension in member C.

1.9 Referring to fig 1.6. If the resultant of the two forces is parallel to the axis of the car, find the value of α for which the tension T_2 is minimum. What is the corresponding value of T_2?

NUMBER: 2

CLUSTER: CONSTRUCTION

AREA: CIVIL ENGINEERING

TITLE: 2 + 3 + 6 = 7

Rationale:

The world of engineering is not a two-dimensional world. It's three-dimensional. This package extends the ideas of the last package from a plane into space. The arithmetic gets more involved, which makes the use of the i, j, k, λ vectors more important.

Behavioral Objective:

Given some problems to solve involving the finding of the resultant of some forces in space, you will be able to find these resultants in 60% of the problems. (All tests are open book. You will also be given a grade for specific homework problems that are asked for).

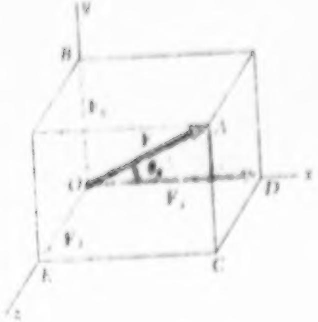
Pre-test:

Page 47 - 49 in the text. Problems 2.67, 2.75, 2.81.

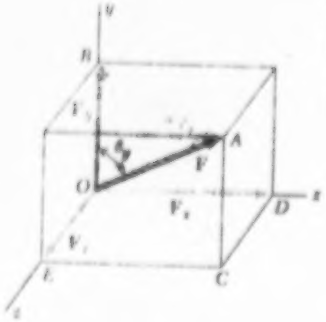
Information Sources:

Text book	"Vector Mechanics for Engineers"
Data Brief # 1	"Pythagoreus: unit vectors"
Data Brief # 2	"Equilibrium"

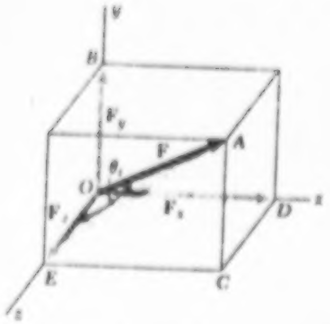
Data Brief # 1



$F_x = F\cos\theta_x$



$F_y = F\cos\theta_y$



$F_z = F\cos\theta_z$

Use any of the above figures to follow thru the explanation below.

$OC^2 + AC^2 = OA^2 = F^2$ Pythagorean theorem

$OD^2 + DC^2 = OC^2$ " "

$OD^2 + DC^2 + AC^2 = F^2$ Substitution

$$F = \sqrt{OD^2 + DC^2 + AC^2}$$

$F_x = OD$

$F_z = DC$

$F_y = AC$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$F = F_x i + F_y j + F_z k$

$F = (F\cos\theta_x)i + (F\cos\theta_y)j + (F\cos\theta_z)k$

$F = F(\cos\theta_x i + \cos\theta_y j + \cos\theta_z k)$

$F = F(\lambda)$ where $\lambda = \cos\theta_x i + \cos\theta_y j + \cos\theta_z k$

If F describes the magnitude of F, then λ must describe the direction of F, and it has the value of 1. Thus it is called the unit vector.

Example. A force F has the components $F_x = 20$ lb, $F_y = -30$ lb, $F_z = 60$ lb. Determine its magnitude F and the angles θ_x, θ_y, θ_z. it forms with the axes of coordinates.

$F = 20^2 + (-30)^2 + 60^2$

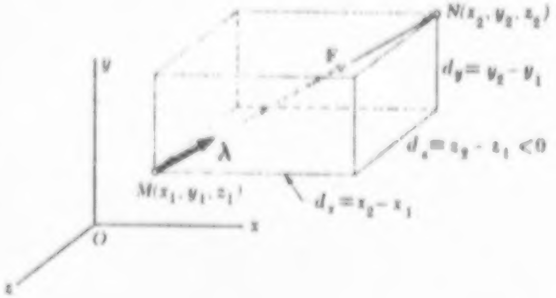
$F = 70$ lb.

$F_x = F\cos\theta_x$

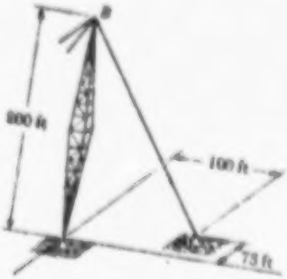
$20 = 70\cos\theta_x$

$\cos\theta_x = 20/70 = 73.4°$

A force is defined by its magnitude and two points on its line of action. (see picture below)



A tower guy wire is anchored by means of a bolt at A. The tension in the wire is 25oo lbs. Determine (a) the components F_x, F_y, F_z of the force acting on the bolt, (b) the angles θ_x, θ_y, θ_z defining the direction of the force.



$F_x = F\cos\theta_x$

$= 2500\cos\theta_x$

$\cos\theta_x = d_x / d$

$d = d_x^2 + d_y^2 + d_z^2 = 236$ ft.

$\cos\theta_x = -100/236$

$F_x = 2500(-100/236) = -1060$ lb.

You find the remaining unknowns and show your work to the teacher before going on.

$F = (2500\text{ lb})\lambda$

Data Brief # 2

If a body is in equilibrium, the sum of all the forces acting on it is zero. That means the sum of the forces in the x direction is zero, because if it isn't than that body is going to move, which means it is not in equilibrium.

Example. A 500 lb weight is hung by means of two cables AB and AC, which are attached as shown. A horizontal force P perpendicular to the wall holds the weight in the position shown. Determine the magnitude of P and the tension in each cable.

$$\overrightarrow{T_{AB}} = B$$

$$\overrightarrow{T_{AC}} = C$$

$$P + W + B + C = 0 \qquad P = Pi$$

$$W = -500j$$

$$Pi + (-500j) + B\lambda_B + C\lambda_C = 0$$

$$\lambda_B = \cos\theta_{x_B} i + \cos\theta_{y_B} j + \cos\theta_{z_B} k$$

$$= \frac{-3\,i}{\sqrt{-3^2 + 25^2 + 20^2}} + \frac{25j}{\sqrt{}} + \frac{20k}{\sqrt{}}$$

$$\lambda_B = -.0932i + 0.776j + .621k$$

$$\vec{B} = B\lambda_B = -.0932Bi + .776Bj + .621Bk$$

$$\vec{C} = -.0845Ci + .704Cj - .704Ck$$

The sum of all the forces is zero!!!!!!

$$0 = Pi - 500j + (-.0932Bi + .776Bj + .621Bk) +$$
$$(-.0845Ci + .704Cj - .704Ck)$$

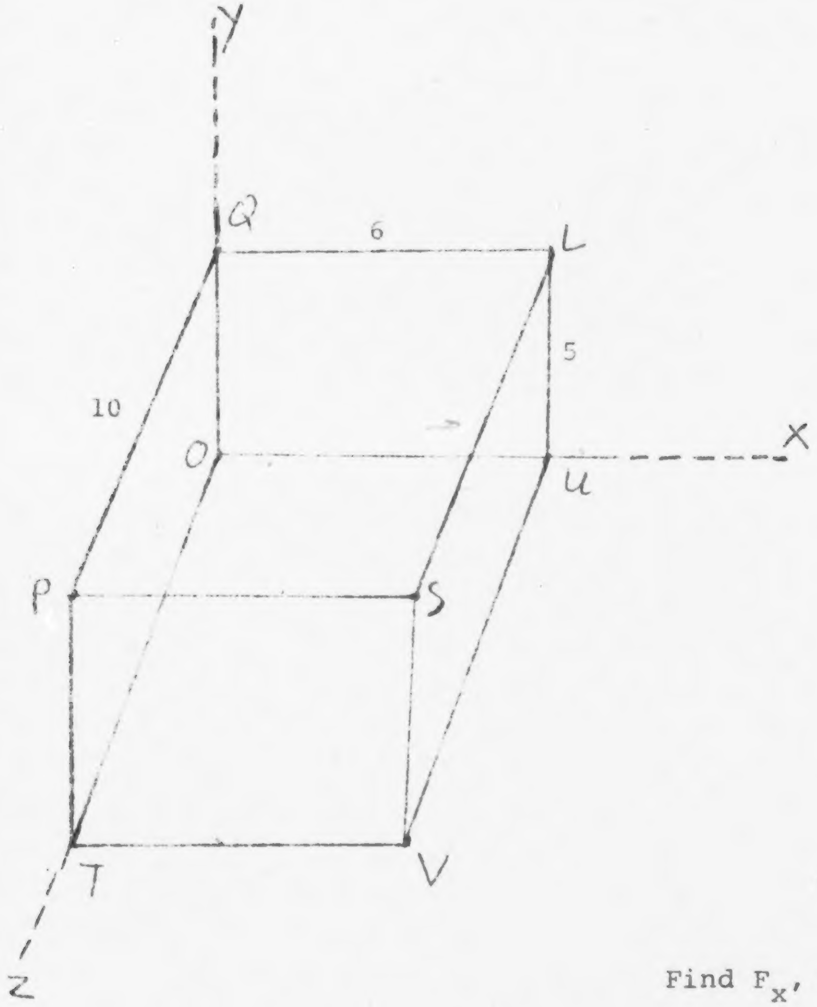
$$0 = (-.0932B - .0845C + P)i + (.766C + .704C - 500)j +$$
$$(.621B - .704C)k$$

The sum of the forces in the i direction = 0. Also in the j and k direction the sum of the forces = 0.

$$-.0932B - .0845C + P = 0$$
$$.776B + .704C - 500 = 0$$
$$.621B - .704C = 0$$

Result $P = 60$ lb $B = 358$ lb $C = 316$ lb

Activity # 1



F = 300 LBS.

Find F_x, F_y, F_z, θ_x, θ_y, θ_z

If the line of action for F is
1. OS
2. OL
3. OU
4. TL
5. VQ

if the line of action is from T to the midpoint of line
6. LS
7. UV
8. QL

if the line of action is from the midpoint of OT to the midpoint of QL.

Determine the magnitude F of $\bar{F}$ and the angles θ_x, θ_y, θ_z it forms with the x,y,z axes:

If $\bar{F}$ = 1. $20i - 30j + 60k$

2. $-25i + 10j - 35k$

3. $5i - 14j + 22k$

4. $15i - 22k$

5. $-14i + 8j - 40k$

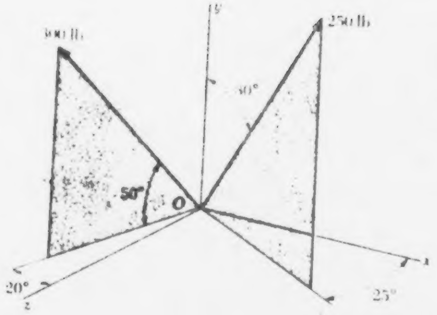


Fig 1.1

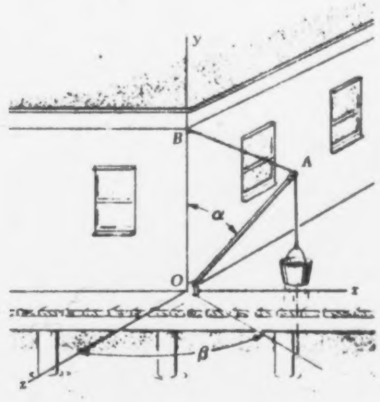


Fig 1.2

1.1 Determine the (a) x, y, z components of the 250 lb force, (b) angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes. Do the same for the 300 lb. force.

1.2 A force of 500 lb, directed along the axis of the boom from A to O, is exerted on the support at O. Knowing that $\alpha = 35°$ and $\beta = 65°$, determine (a) the x, y, and z components of the force exerted at O, (b) the angles θ_x, θ_y, and θ_z for the force exerted at O.

1.3 A 400 lb force acts at the origin in a direction defined by the angles $\theta_y = 80°$ and $\theta_z = 46°$. It is also known that the x component of the force is positive. Determine the value of θ_x and the components of the force.

1.4 A precast-concrete wall section is temporarily held by the cables shown. If the tension in cable AB is 700 lbs, determine the components of the force exerted on the wall at A.

1.5 Knowing that the tension in the cable BC is 900 lb, determine the components of the force exerted on the wall at point C

1.6 Determine the magnitude and direction of the force = 150i -75j + 200k.

1.7 Do the same for the force = -150i - 350j + 200k

1.8 The tension in AB is 5200 lb, and in AC it is 3500 lb. Find the resultant of these two forces on point A.

1.9 If AC = 7000, determine the tensions in AB and AD such that the resultant of these three forces at A is vertical.

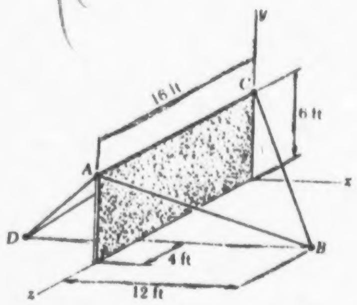


FIG 1.4, 1.5

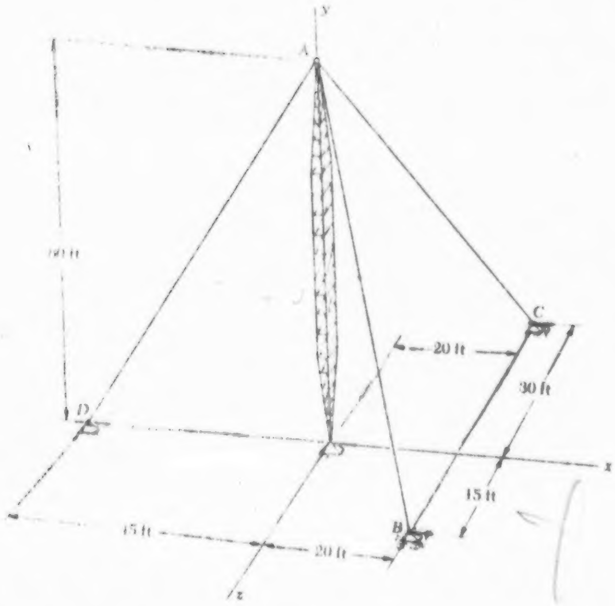


FIG. 1.8, 1.9.

Activity # 2

In the text page 47 - 50

# 2.66, 2.68, 2.69, 2.70, 2.72, 2.73, 2.74, 2.80, 2.87.

If you follow the format of Data Brief # 2, you should be able to work the problems. Get used to using i, j, k, λ.

NUMBER: 3

CLUSTER: CONSTRUCTION

AREA: CIVIL ENGINEERING

TITLE: JUST A MOMENT!!

Rationale:

The idea of a moment is fairly easy to understand in a 2-dimensional system. In a 3-D system the mathematics is too complicated to work through without vectors. With vectors the idea of a moment in space is more easily understood. And determinants greatly simplify the arithmetic.

Behavioral Objective:

Given some problems to solve involving the finding of moments, you will be able to find the moments by using the cross product, (for 3-D problems you must use determinants), with 60% accuracy.

Pre-test:

Page 66 - 68 # 3.7

3.13a
3.20(Let P = 900 lbs.) } you must use determinants

Information Sources:

Text book	"Vector Mechanis for Engineers"
Data Brief # 1	"Moments"
Data Brief # 2	"Cross products & Determinants"
Data Brief # 3	"Cross Products"

Data Brief # 1

Momemts:

The Line of action of a moment represents the axis about which the body tends to rotate.

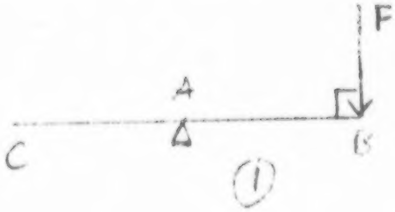
The sense of a moment characterizes the sense of rotation that a force tends to impart to a rigid body. An observer located at the top of the arrow of the moment vector will see a ccw rotation.

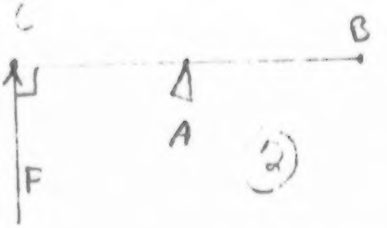
The magnitude of a moment measures the tendency of a force to impart a rotational motion to a body.

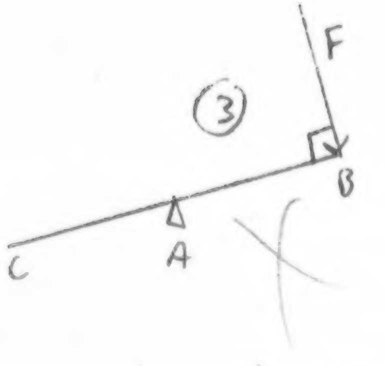
Data Brief # 1 (continued)

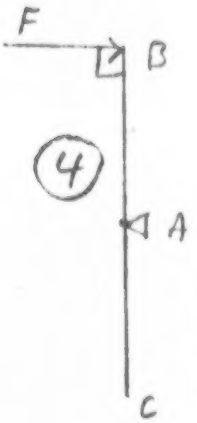
F = 10 lb.

AB = AC = 4'









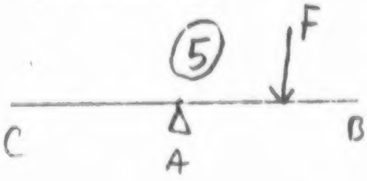
In each drawing above, (massless) bar BC will tend to rotate at the same speed for the same number of revolutions.

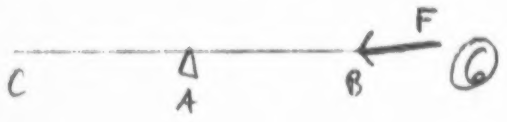
In the picture to the right, the bar will not rotate as fast or as much as in the first four drawings.

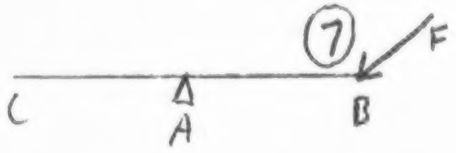
In drawing #6 the bar won't rotate at all.

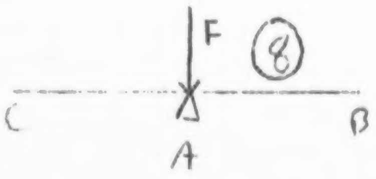
In drawing #7 the bar will rotate slower and fewer number of times than in the first four drawings

In drawing # 8 the bar won't rotate at all.









To change the rotational characteristics of a body, you can

Apply the force different distances from the pivot point. #5,8

Change the angle of application of the force. #6,7

$M_O M_o$

Change the magnitude of the force. In drawing 1 thru 4, change the force to 100 lbs.

M_{oo}

The magnitude of a moment (M_O) measures the tendency of the force (F) to impart a rotational motion to a rigid body when the body is attached at O ($_O$).

$$M_O = Fd$$

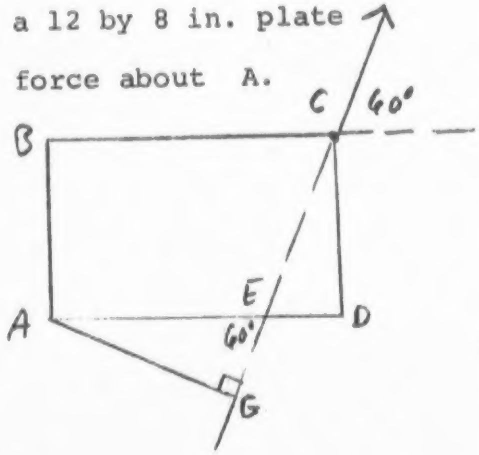
where d is the perpendicular distance from O to the line of action of F.

Read thoroughly sample problem 3.1, p. 63 of the text.

The following sample problem uses the picture at the top of page 64.

A 10 lb. force acts on the corner of a 12 by 8 in. plate as shown. Determine the moment of the force about A.

Extend the line of force so that you can draw a perpendicular line from it to point A.



$$\sin 60^\circ = \frac{AG}{AE} = \frac{d}{AE}$$

$$d = AE \sin 60^\circ = AE\frac{\sqrt{3}}{2}$$

$$AE = 12 - ED$$

$$\tan 60^\circ = \frac{CD}{ED} = \frac{8}{ED}$$

$$ED = \frac{8}{\tan 60^\circ} = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$$

$$AE = 12 - \frac{8\sqrt{3}}{3} = \frac{36-8\sqrt{3}}{3}$$

$$d = \left(\frac{36-8\sqrt{3}}{3}\right)\left(\frac{\sqrt{3}}{2}\right) = 6\sqrt{3}-4$$

$$M_A = Fd = 10(6\sqrt{3}-4) = 64 \text{ lb-in}$$

$$\vec{M}_A = 64 \text{ lb in} \circlearrowleft$$

Look at sample problem # 3.3. This is a different method than the one described above. Use this method to do sample problem # 3.2, and see if you come up with the same answer.

Data Brief # 2

$$V = P \times Q$$

V is the cross product of P and Q. This is vector multiplication and you may not substitute a dot for the cross. You may not substitute parentheses either. The cross product is the cross product and nothing else.

The magnitude V, is found by the following equation:

$$V = PQ\sin\theta$$

where θ is the angle between the two vectors. (Fig. 3.6, p. 55)

The line of action (direction) of V is perpendicular to the plane containing containing P and Q.

You always rotate the first letter into the second letter. This operatio-n is not commutative. Looking at the picture on p. 55, if you curl your fingers in the direction P must rotate, your thumb will point upward. Thus the right hand rule is defined. If your fingers curl in the direction of rotation, your thumb points in the direction of the vector representing the moment.

$$V = P \times Q = (P_x i + P_y j + P_z k) \times (Q_x i + Q_y j + Q_z k)$$

$i \times i = 0$
$i \times j = k$
$i \times k = -j$

$j \times i = -k$
$j \times j = 0$
$j \times k = i$

$k \times i = j$
$k \times j = -i$
$k \times k = 0$

If you now perform the above multiplication, the result will be the equation 3.8 on p. 58. Of course there is an easier way to do the multiplication, if the vector quantity is set up in the form of a determinant

To evaluate a determinant $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ copy the first two columns after the determinant $\left|\begin{array}{ccc|cc} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{array}\right.$

Now multiply each element in a diagonal from top to bottom and subtract from this the sum of the product of the elements in the diagonals going from the bottom to the top.

The determinant is $aei + bfg + cdh -(ceg + afh + bdi)$

which can also be written as

$$aei - afh = a(ei - fh)$$

$$+$$

$$bfg - bdi = b(fg - di) = -b(di - fg)$$

$$+$$

$$cdh - ceg = c(dh - eg)$$

example. Evaluate $\begin{vmatrix} i & j & k \\ 2 & 33 & 1 \\ 0 & 4 & 0 \end{vmatrix} = i \begin{vmatrix} 33 & 1 \\ 4 & 0 \end{vmatrix} + -j \begin{vmatrix} 2 & 1 \\ 0 & 0 \end{vmatrix} + k \begin{vmatrix} 2 & 33 \\ 0 & 4 \end{vmatrix}$

$= i(-4) - j(0) + k(8)$

$= -4i + 8k$

Rules for making the evaluation of determinants easier.

If you interchange two rows (or columns) you must change the sign of the determinant.

You may factor a constant out of any row or column.

You may use the first column exactly as you use the first row when evaluating the determinant.

Examples.
$$\begin{vmatrix} 2 & 3 & 4 \\ 1 & 5 & 6 \\ 3 & -1 & -1 \end{vmatrix} = -\begin{vmatrix} 2 & 3 & 4 \\ 3 & -1 & -1 \\ 1 & 5 & 6 \end{vmatrix}$$

$$\begin{vmatrix} 100 & 200 & -100 \\ 2 & 3 & -1 \\ 4 & 0 & 15 \end{vmatrix} = 100\begin{vmatrix} 1 & 2 & -1 \\ 2 & 3 & -1 \\ 4 & 0 & 15 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 1 & -1 \\ 0 & 4 & 2 \\ 1 & -1 & 3 \end{vmatrix} = 2[12-(-2)] -1(0-2) - 1(-4) = 34$$

Using the first column instead of the first row you get

$$2[12 -(-2)] -0(\quad) + 1(2 -(-4)) = 34$$

Data Brief # 3

In finding vector cross products, the commutative and associative laws do not hold and the distributive law does hold.

Go thru example 3.2 on page 64. Use this method to solve all the problems of only two dimensions.

Starting with problem 3.15, use determinants. Refer to sample problem 3.4 p. 65 whenever you get stuck.

Activity # 1

Use only the methods discussed so farto solve the following problems.

P. 66 of the text.

3.1 The force of 120 lb is not perpendicular to the line AB. If it were you would be done. What part of the 120 lb is perpendicular to the line AB?

3.2, 3.3, 3.4

3.5 Just find the moment of the 100 lb. force about A.

3.6, 3.7

3.8 Just find the moment of the 260 lb force about A.

Activity # 2

Evaluate the following determinants.

1. $$\begin{vmatrix} 2 & 5 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix}$$

2. $$\begin{vmatrix} 3 & 6 & 9 \\ 0 & 5 & 5 \\ 2 & 1 & 0 \end{vmatrix}$$

3. $$\begin{vmatrix} 8 & 8 & 2 \\ 0 & 6 & 0 \\ -4 & -4 & -1 \end{vmatrix}$$

4. $$\begin{vmatrix} 2 & 0 & 1 \\ 0 & 1 & 4 \\ \frac{20}{3} & \frac{30}{3} & \frac{-50}{3} \end{vmatrix}$$

5. $$\begin{vmatrix} 2 & 1 & 3 \\ 0 & 1 & 3 \\ 5 & 4 & 0 \end{vmatrix}$$

6. $$\begin{vmatrix} i & j & k \\ 6 & 0 & 3 \\ -1 & 2 & -1 \end{vmatrix}$$

For each problem below, find R x F

7. R = 3i -4j + 2k F = 210i -400j -160k

R = 2i -j -k F = 8i + 6j + jk

R = 4i -7k F = 600i -200j + 300k

R = 8i - 12j + 2k F = 120i + 120j - 120k

R = 3i + 3j + 3k F = 10i + 10j + 10k

Activity # 3

Do all the problems in activity # 1 using the cross product method.

P. 67-8 3.15, 3.16, 3.17, 3.18, 3.19, 3.20, 3.21, 3.22

NUMBER:	4
CLUSTER:	CONSTRUCTION
AREA:	CIVIL ENGINEERING
TITLE:	DOT YOUR I'S, J'S, AND K'S

Rationale:

The concept of the "dot product" is introduced in this package, to allow you to solve a greater variety of problems involving moments. Again, determinants are used to simplify the arithmetic.

Behavioral Objective:

Given some problems that require you to use the mixed triple products of three vectors, you will be able to solve those problems with 60% accuracy. (Open book test).

Pre-test:

Refer to fig 3.27, 3.28, p. 75.

Let the bottom of the tower be labelled "O".

If the tension in cable AB is 60 lbs from A to B, find the moment of this force about the line joining O to C.

First use R from O to A. Then do the problem again using R from C to A.

Information Sources:

Text book

Data Brief # 1	"Projecting a vector"
Data Brief # 2	"Dot product"
Data Brief # 3	"Lambda again"
Data Brief # 4	"Moments about axes"

Data Brief # 1

The cross product is a vector product

$$V = P \times Q$$

V is a vector.

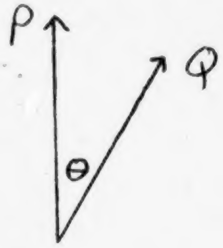
P·Q is a scalar product. It is called the dot product.

$$P \cdot Q = PQ\cos\theta$$

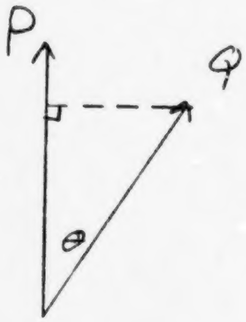
where θ is the angle formed by the two vectors. The dot product has magnitude only, no direction.

One of the uses of the dot product is to find the product of a vector and the projection of another vector on the first vector.

Example. What is the product of P and that part of Q which acts along the line of action of P.



The component of Q acting along the line of action of P, has to be less than the value of Q. You have to split Q into two components, so the other component must be ⊥ to P. Thus the right triangle is formed with Q as the hypotenus.



$$P \cdot Q = PQ\cos\theta$$

The projection of Q onto P is Qcosθ

The projection of P onto Q is Pcosθ

For this last projection you would have to redraw a ⊥ from P to Q.

Data Brief # 2

$$P \cdot P = (P)(P)\cos 0° = P^2(1) = P^2$$

$$i \cdot i = (i)(i)\cos 0° = i^2(1) = i^2$$

But remember that i is a vector of unit length. Therefore

$$i^2 = 1^2 = 1$$

This should help you to understand the table (3.26) on page 69. Do you understand why $i \cdot j = 0$?

$$P \cdot Q = (P_x i + P_y j + P_z k) \cdot (Q_x i + Q_y j + Q_z k)$$

$$P \cdot Q = P_x Q_x + P_y Q_y + P_z Q_z \qquad (3.27)$$

$$P \cdot Q = PQ\cos\theta$$

$$\cos\theta = \frac{P_x Q_x + P_y Q_y + P_z Q_z}{PQ} \qquad (3.29)$$

EXAMPLE. Compute the dot product of the two vectors P and Q and find the angle between them.

$$P = 2i + 3j - 6k \qquad Q = -i + 3j - 2k$$

(Using equation 3.27) $P \cdot Q = 2(-1) + 3(3) + (-6)(-2) = 19$

$P = \sqrt{2^2 + 3^2 + -6^2}$ $\qquad Q = \sqrt{14}$

Now use equation 3.29

Data Brief # 3

Remember λ ?????????

$Q = Q\lambda$ Q is the magnitude only

λ is the direction only with a magnitude of one.

P. 70 EQ. 3.31 + 3.32

$$P_{OL} = \frac{\vec{P} \cdot \vec{Q}}{Q}$$

$$\vec{Q} = Q\lambda$$

$$P_{OL} = \frac{\vec{P} \cdot (Q\lambda)}{Q} = \vec{P} \cdot \lambda$$

$$\lambda = \cos\theta_x i + \cos\theta_y j + \cos\theta_z k$$

$$\lambda = \frac{d_x}{d} i + \frac{d_y}{d} j + \frac{d_z}{d} k$$

$$\lambda = \frac{d_x i + d_y j + d_z k}{d}$$

P. 75 fig. 3.27, 3.28

Suppose there is a force of 910 lb whose line of action is from A to C. Find the projection of this force on AB.

$$\vec{F} = 910 \text{ lb}$$

$$F_{AB} \text{ (SCALAR)} = \vec{F} \cdot \lambda_{AB}$$

$$= (F \lambda_{AC}) \cdot \lambda_{AB}$$

$$= 910\left(\frac{2}{7} i - \frac{6}{7} j - \frac{3}{7} k\right) \cdot \left(\frac{4}{13} i - \frac{12}{13} j + \frac{3}{13} k\right)$$

$$= \frac{910}{7 \cdot 13} (2i - 6j - 3k)(4i - 12j + 3k)$$

$$= 10(8 + 72 - 9) = 710 \text{ lbs}$$

$$F_{AB} = F \cos\theta \quad \text{(EQ 3.30 P. 70)}$$

$$710 = 910 \cos\theta$$

$$\cos\theta = \frac{710}{910} = .7802$$

$$\theta \doteq 39^\circ$$

Data Brief # 4

The dot product is a vector times a vector. with a scalar as athe product.

$\lambda \cdot V$ is a dot product.

It will tell you how much of V is acting in the direction of the unit vector λ.

Suppose $V = P \times Q$

Then V is a vector perpendicular to the plane which contains P and Q.

$\lambda \cdot V$ will tell you how much of the vector V is acting in the λ direction.

Fig 3.25 p. 72 $R \times F$ produces a moment M_o as pictured.

$R \times F$ will produce a maximum rotation about an axis thru O in the direction of M_o. But $R \times F$ will produce a smaller rotation about the line OL, smaller because OL is not perpendicular to $R \times F$.

$M_{OL} = M_o \cos\theta$ θ is the angle formed by M_o & OL

$$M_{OL} = \lambda \cdot M_{OL}$$

$$= \lambda \cdot (R \times F)$$

Determinants are used to compute these products. See equation 3.40, p. 72.

Activity # 1

For each problem below, find the projection of P omto Q (P_{ac}), of Q onto P (Q_{ab}), and the dot product.

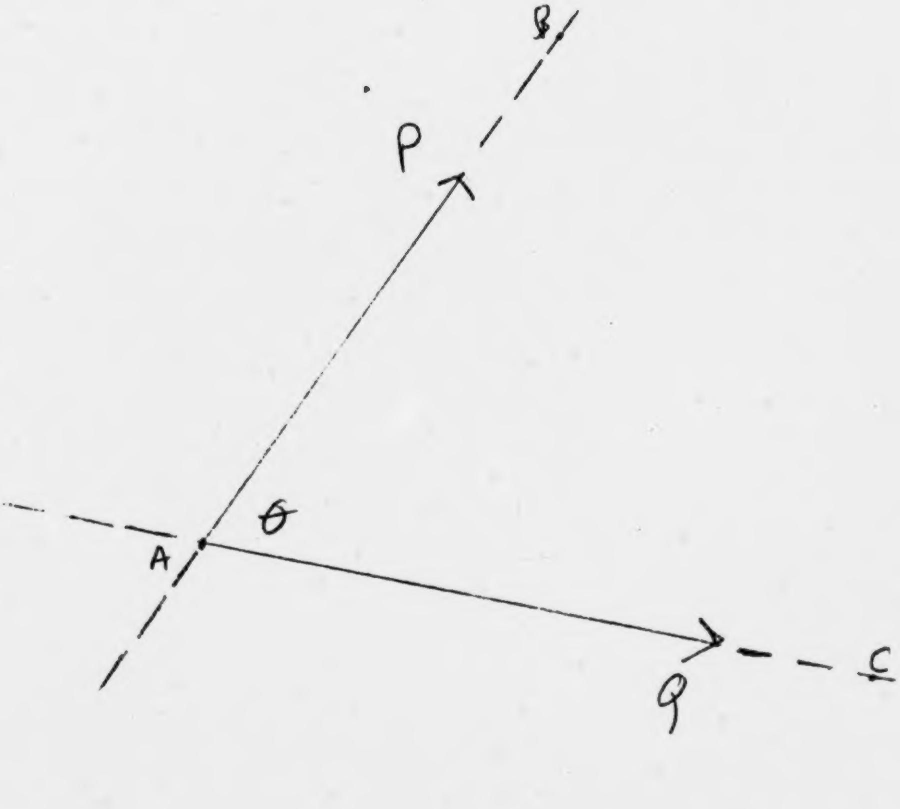
1. P = 100
Q = 120
θ = 30°

2. P = 40
Q = 80
θ = 45°

3. P = 200
Q = 150
θ = 0°

4. P = 250
Q = 450
θ = 90°

5. P = 250
Q = 250
θ = 60°



Activity # 2

1. Use equation 3.27 for problem 1, activity #1, so that you can see that this equation works in a plane as well as in space.

2. # 3.25 p. 75

3. Do the same as # 2 with $P = 4i - 3j - 5k$

$Q = -i + 3j - 2k$

$S = 4i - 3j - k$

4. Find the angle between each pair of vectors according to the table below. (Using problems 2 and 3 above).

vectors	problem 1	problem 2
	$\theta =$	$\theta =$
P & Q		
P & S		
Q & S		

Activity # 3

1. Use Fig 3.27, 3.28 p. 75.

There is a force of 195 lbs from A to D. Find the projection of this force on AB. Find the angle between these two cables. (ans. 108 lbs, 56.4°)

2. Problem 3.29
3. problem 3.30
4. problem 3.31
5. problem 3.32
6. p. 67 fig 3.15, 3.16.

The tension in cable BC acting from B to C is 700 lb. Determine the projection of this force on cable AB.

Activity # 4

Use fig 3.40 p. 76

For all the problems using fig 3.40 p. 76

$$F = -80j + 60k$$

1. Determine the moment about line AC.

   (a) let R be the distance from C to F

$$\lambda_{AC} = \frac{12}{\sqrt{481}} i + \frac{16}{\sqrt{481}} j$$

$$\vec{r} = -12k$$

$$M_{AC} = \begin{vmatrix} \frac{12}{\sqrt{481}} & \frac{16}{\sqrt{481}} & 0 \\ 0 & 0 & -12 \\ 0 & -80 & 60 \end{vmatrix} = ?$$

   (b) Now let R be the distance from A to F

$$M_{AC} = \begin{vmatrix} \frac{12}{\sqrt{481}} & \frac{16}{\sqrt{481}} & 0 \\ 15 & 16 & -12 \\ 0 & -80 & 60 \end{vmatrix} = ?$$

Both moments above should come out the same. What does this tell you about picking R. Which part above was the easiest?

2. Determine the moment about line DE

(a) let R be the distance from D to F.

$$\lambda_{DE} = \frac{15}{25}i - \frac{16}{25}j - \frac{12}{25}k$$

$$\vec{r} = 15i - 12k$$

$$M_{DE} = \begin{vmatrix} \frac{15}{25} & -\frac{16}{25} & -\frac{12}{25} \\ 15 & 0 & -12 \\ 0 & -80 & 60 \end{vmatrix} = ?$$

(b) Now let R be the distance from E to F

$$M_{DE} = \begin{vmatrix} \frac{15}{25} & \frac{-16}{25} & \frac{-12}{25} \\ 0 & +16 & 0 \\ 0 & -80 & 60 \end{vmatrix} =$$

3. Determine the moment about line GB.

   (a) let R be the distance from B to F.

   (b) let R be the distance from G to F

4. Determine the moment about line AF. What is the length of the R vector? What is the moment? (No calculations are necessary).

Problems p. 76 - 77

3.35, 3.37, 3.39, 3.40, 3.41,3.42, 3.43, 3.44, 3.47

CLUSTER: MANUFACTURING

AREA: DESIGN

TITLE: THE FRUSTRATED FRUSTRUM

BY: JIM MADEHEIM
SAHUARITA HIGH SCHOOL
MATH DEPARTMENT

Rationale:

There is a process for finding the volume of a frustrum of a cone, given the formula for the volume of a cone. This process results in another formula, a more complicated one. The process is used in this package as a demonstration only.

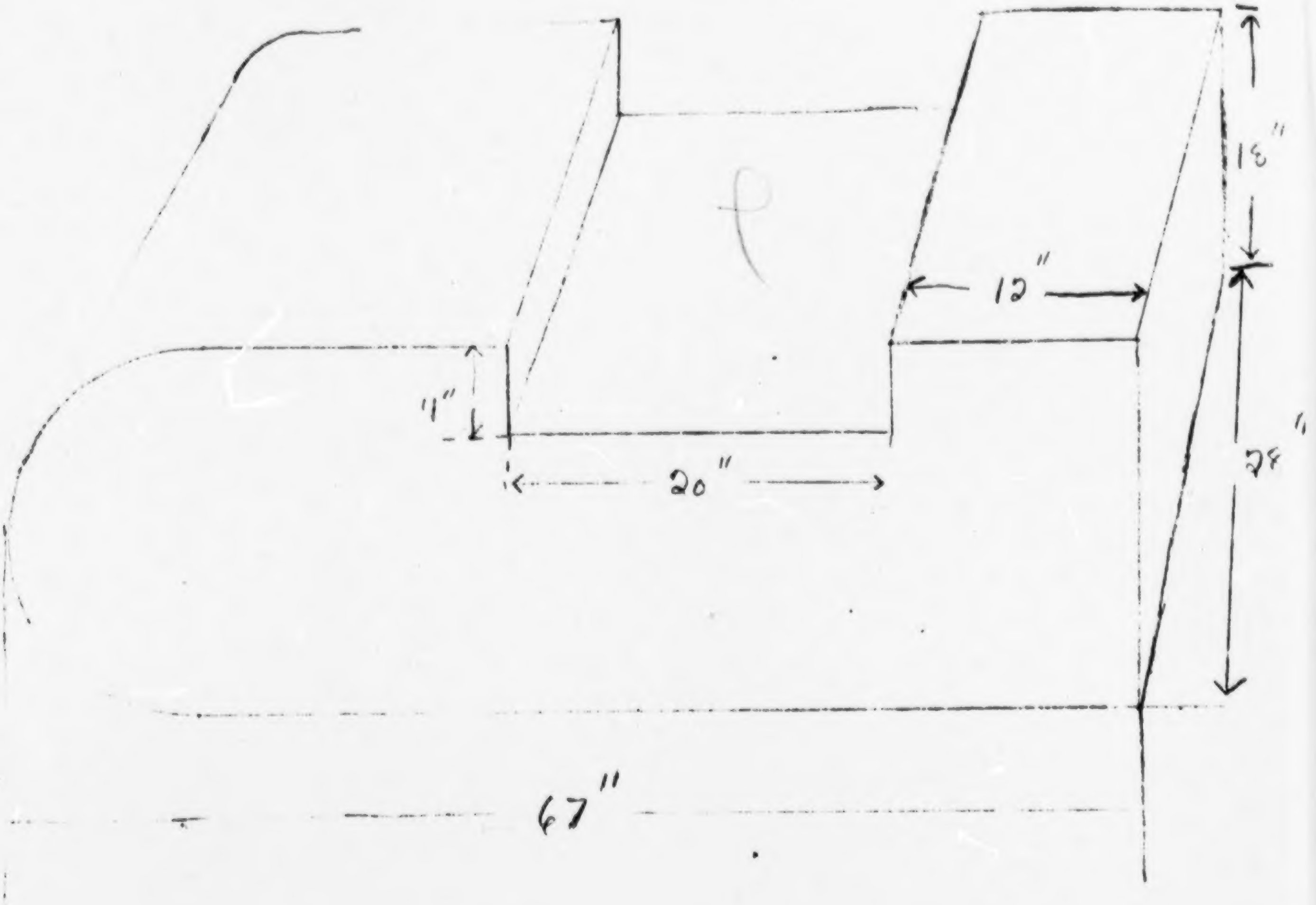
After the new formula has been developed once, and recorded somewhere, there is no reason why you shouldn't use the new formula

This package will give you experience in using area and volume formulas as needed by the machine trades. Part of each assignment will be looking up the formula in a reference book.

Behavioral objective:

Given access to reference materials, and given a problem to solve that involves finding the volume of an object pertaining to the machine trades, you will be able to find that volume with four digit accuracy, 75% of the time.

Pre-evaluation:



1. Find the weight of the above brass casting.

2. Double every dimension. What is the new weight?

You have only 15 minutes to complete the above test!!!!!!

Information Sources:

1. Read Data Brief # 1 "Changing a formulas form."
2. Read Data Brief # 2 "Weights. Volume formulas."

Data Brief # 1

It is sometimes convenient to change the form of a formula and solve it for a variable different than that in the original form.

Example:

Given: $C = 2\pi r$

Solve for r.

$$C = 2\pi r$$

$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$$

$$\frac{C}{2\pi} = r$$

Given: $K = \frac{1}{2}h(s_1 + s_2)$

Solve for s_2.

$$K = \frac{1}{2} h (s_1 + s_2)$$

$$\frac{2K}{h} = \frac{2}{h}\left(\frac{1}{2}h\right)(s_1 + s_2)$$

$$\frac{2K}{h} = s_1 + s_2$$

$$\frac{2K}{h} - s_1 = s_1 + s_2 - s_1$$

$$\frac{2K}{h} - s_1 = s_2$$

Given: $A = \pi r^2$

Solve for r.

$$A = \pi r^2$$

$$\frac{A}{\pi} = \frac{\pi}{\pi} r^2$$

$$\frac{A}{\pi} = r^2$$

$$\sqrt{\frac{A}{\pi}} = \sqrt{r^2} = r$$

Data Brief # 2

See the teacher for a demonstration on finding the formulas for frustrums.

Every problem is Activity # 2 can be solved by substituting known quantities into a formula and grinding out an answer. Part of the assignment for each problem is the decision on which formula to use and the finding of the formula in some reference book.

	Wt/cu.in.	Wt/cu.ft.
Steel	0.283	489.0
Cast iron	0.2600	449.2
Bronze	0.3195	552.2
Brass	0.3018	521.7
Wrought Iron	o.2834	489.8

Data Brief # 2 (cont.)

The following information is needed starting with problem # 81.

A solid ring is made with a piece of stock bent into circular form, the cross section of which may be either a circle, square, triangle, or any other shape. The length of stock required equals the length of the rings center line.

Activity # 1

1. Solve the formula used in finding the total surface area of a -

right circular cone, for h.

sphere, for r.

frustrum of a regular pyramid, for s (slant height).

2. Solve the formula used in finding the volume of a -

Sphere, for r.

right cylinder for r.

right cylinder for h.

non-right cylinder for s (slant height).

frustrum of a right circular cone for the radius of the smaller base (R_1).

frustrum of a pyramid for the area of the larger base (B_2).

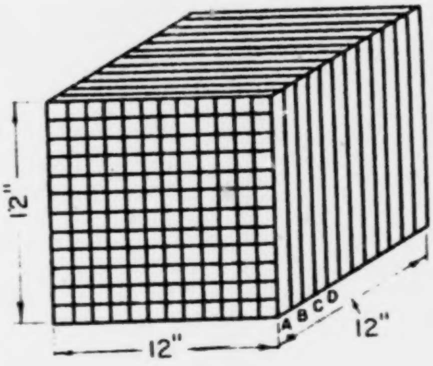
3. Solve the formula for finding the area of a sector of a circle for r.

Do the same for the area of a segment of a circle.

4. Solve $s = \sqrt{(R_1 - R_2)^2 + h^2}$ for R_2

5. Solve $V = \frac{1}{6}\pi h(3a^2 + h^2)$ for a.

Activity # 2



1. What is the area of the front of the prism shown? ______

2. If a slice A, 1" thick. is cut from the prism, how many 1" cube blocks are in this slice? ______

3. If two slices A and B are cut from the prism, find the number of cubic inches in the two pieces. ______

4. How many cubic inches are contained in a piece 3" thick which is cut from this prism? ______

5. How many cu. inches in the entire prismshown? ______

6. What is the volume, in cu. in. if a 1' cube? ______

7. What is the area of the end piece of 3/4" x 1 1/2" steel bar? ______

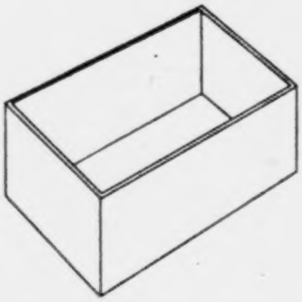
8. What is the volume of a piece of 3/4" c 1 1/2" steel bar 1" long? ______

9. Find the volume of a piece 5/8" x 2" steel bar 7 3/4" long. ______

10. Find the weight of a piece of steel 9/16" x 2" x 4'-2". ______

11. What length must a piece of 2" square steel bar be to weigh exactly 5 lbs? __________

12. The weight of 15 pieces of 3/8" x 1 1/4" steel bar of equal length is 17 1/2 lbs. What is the length of stock used, if 1/16" is allowed for each saw cut? __________



13. A rectangular box similar to sketch, is made of 1/8" sheet steel. Find the weight of the box if outside dimensions are 4' x 3.4' x 2.6', and the steel used weighs .29 lb per cu. in. Box is to have bottom but no top. __________

14. 180 sq. ft. of zinc are required to line the bottom and sides of a cubical box. How many cu. ft. of water will it hold? __________

15. Find the cost at 52¢ per lb. for sheet copper to line the bottom and sides of a cubical box 7' on an edge, if the copper sheets weigh 12 oz. per sq. ft. __________

16. How many gallons of oil will the box in problem 15 hold? __________

17. Find the weight of a piece of steel 1/16" thick, 2'

wide and 4' long. ______________

18. Find the weight of a piece of steel .432" thick, 2'-3" wide and 7' long. ______________

19. Find the weight of a piece of steel , No. 11 gage (U.S. Std.), 1'-1" wide and 3'-1/2" long. ______________

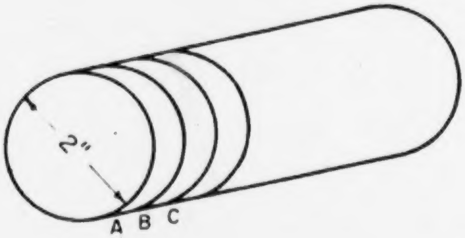
20. A piece of 22 gage steel (U.S. Std.), 4 1/2' long weighs 20 lbs. Find the width of the piece. ______________

21. A piece of sheet steel 2' x 6' weighs 9.14 lbs. Find the thickness to the nearest thousandth. What gage is this?

____

22. A piece of sheet steel 3'-3 1/4" x 7'-6 1/4" weighs 223 lbs. Find its thickness. What gage is this? ______________

23. What is the area of the end of the 2" diameter cylinder shown?



24. If the three slices A, B, and C each 1" long are cut from the cylinder, how many cu. in. do the three contain?

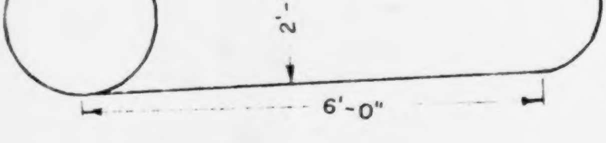
25. Find the volume of a piece of 1 1/4" round stock 3 1/2' long.

26. Find the weight of a steel bar 3/4" in diameter and 20' long.

27. If the volume of a 3" diameter cylinder is 49.48 cu.in., find the length in inches.

28. Find the length of a 1 1/8" diameter cylinder with a volume of 5.467 cu.in.

29. How many sq. ft. of sheet metal are required for each end of the cylindrical tank illustrated?



30. How many sq. ft. of sheet metal are used in a piece 6' long forming the side of the tank, allowing nothing for lap? ____________

31. If the metal is 1/8" thick, what is the volume of metal in the tank? What will it weigh? (steel) ____________

32. How many cubic feet of water will the tank hold? How many gallons? ____________

33. A cylindrical oil tank 3' in diameter and 10' long will contain how many gallons of oil? ____________

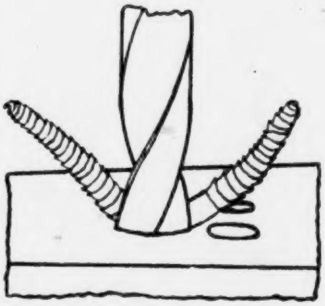
34. Find the weight of a cylindrical tank 2 1/2' in diameter x 4' long, which is made of steel plate 3/16" thick, making no allowances for laps and rivets. ____________

35. A cylindrical tank 3' in diameter and 3' long made of 1/8" thick steel is filled with water. Find the weight of the tank when filled. ____________

36. If the tank in the preceeding problem is painted on the outside, how many sq. ft. of surface must be covered? ____________

37. In drilling soft steel, a 1 9/16" drill makes 37 rpm. with a feed of 1/80". How far does the drill advance in 3 1/2 minutes? ____________

38. Find the number of cu.in. of metal cut away, in the preceeding problem. ____________



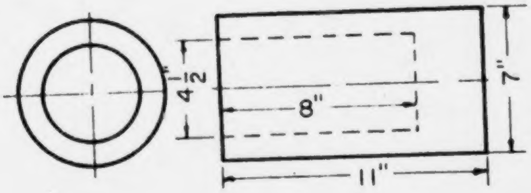
DRILLING

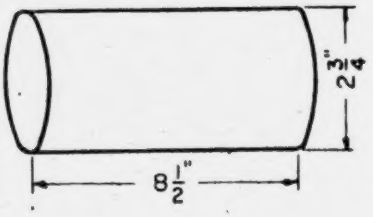
39. A 1 1/2 " drill turns 40 rpm. with a feed of .012" while drilling a rectangular block of steel 2" x 3" x 3". Find the weight of steel removed in 2 minutes and the weight of the remaining block after drilling for 5 minutes.

40. In turning a steel shaft 6" in diameter and 4' long, the cuttign speed was 36' per min. and the feed was .125". If the depth of the cut was .05", find the amount of metal removed. (Cutting done on a lathe)

41. A piece of stock is turned in a lathe from 2 1/4" down to 1 3/4" in diameter. If the piece was 7" long, how many cu. in. of metal were removed?

42. The cylindrical piece of steel as illustrated, is to be carbonized on its lateral surface as part of the heat treating process it is to go through. Find the area to be carbonized. (Picture on the right)



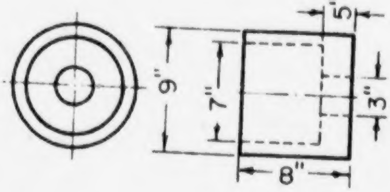


43. Find the weight of the piece shown on the left, if it is made of cast iron.

43. A tank for a hardening bath is 2'-4" in diameter and 2'-8" deep. How many gallons of liquid will be required to fill the tank within 5" of the top?

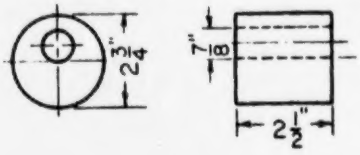
44. A counterweight is made of a cylindrical piect of cast iron 1 7/8" in diameter. How long should it be, if it must weigh 57 lbs? ________________

45. Find the total area of a casting 9" in diameter with a 6 1/2" hole through it. The piece is 7" long. ____________



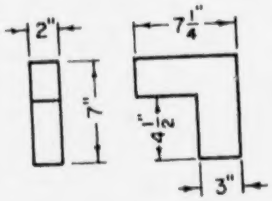
46. What is the volume of the cast iron sleeve, as illustrated above? ____________

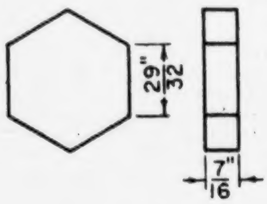
47. How many cubic inches in the brass bushing as shown to the right? ____________



48. A cast iron collar 4" in diameter has a 1 15/16" hole in the center and is 2" long. How many cu. In. of iron make up the collar? ____________

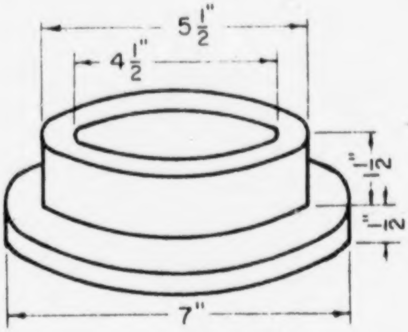
49. How many cu. in. of cast iron are there in the picture on the left? ____________





50. How many cu. in. are there in the brass blank for a hex nut in the picture on the right? ____________

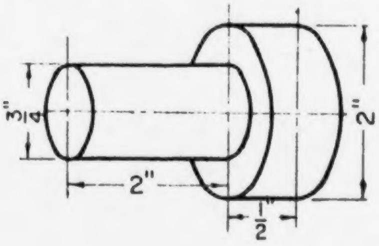
51. What is the volume of metal in the bronze ring as illustrated below? ____________



52. Calculate the cu.In. of metal, per foot of length, in a copper tube whose outside dia. is 2" and inside dia. is 1 1/2". ____________

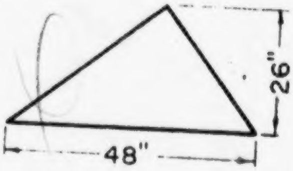
53. Find the net volume of steel plate 1/2" thick, that is 4' x 2 1/4' with 5 holes through it 4" in diameter. ____________

54. The pin in the accompanying sketch below is to be made of steel. Calculate the number of cu. in. in 408 such pins and find their total weight. ____________



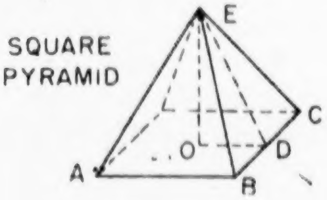
54. A cast iron sleeve, rectangular in form, measures 2 1/2' x 5 1/4' on each end and is 8 1/2' long. There is a cored hole 1' x 4' which extends through its entire length. How many cu. ft. of iron are there in this sleeve? ____________

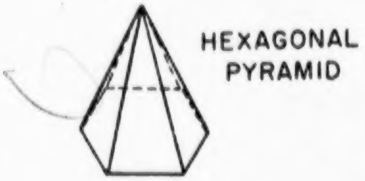
55. How many cu. in. are in the triangular steel plate illustrated at the right if it is 7/16" thick? ___________



56. How many gallons of cutting oil can be stored in a tank the inside measurement of which is 37" x 96" x 118"?___________

57. If the base of a square pyramid, as shown below, is 4" on a side and the height EO is 6", find the slant height ED. ____________





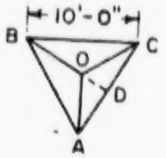
58. Find the area of lateral side BEC of the pyramid. ____________

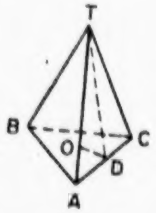
59. Find the lateral area of the pyramid. ____________

60. Multiply the perimeter of the base by 1/2 the slant height. ____________

61. Find the total area of the surface of the square pyramid shown. ____________

62. Find the lateral area of the hexagonal pyramid, as shown, using the same dimensions. ____________





63. Each side of the base of the triangular pyramid shown above is 10' and the altitude is 18'. Find BD. ____________

64. Using the pictures above, how many degrees in angle OCD? What part of OC is OD? ____________ ____________

What part of BD is OD?

What length is OD,

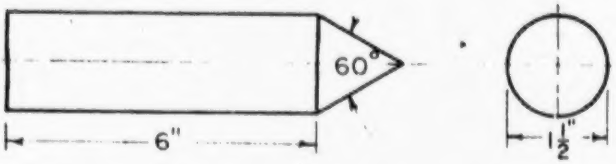
65. Find the slant height TD.

66. Find the length of lateral edge TA.

67. Given a cone with an altitude of 8" and radius of 6", find the slant height.

68. What is the lateral area of the cone in problem 67?

69. What is the total area of the cone in problem 67?



70. Find the overall length of the piece illustrated above.

71. Find the volume of the cylindrical part and of the cone.

72. Find the total area and volume of a cone whose altitude is 12' and whose base diameter is 10'

73. Find the weight of a conical casting of iron 8" in dia. with a slant height of 14".

74. Find the weight of a cast iron ball 1' in dia. __________

75. If the above cast iron ball is coated with lead 1" thick, what is the new weight of the ball? __________

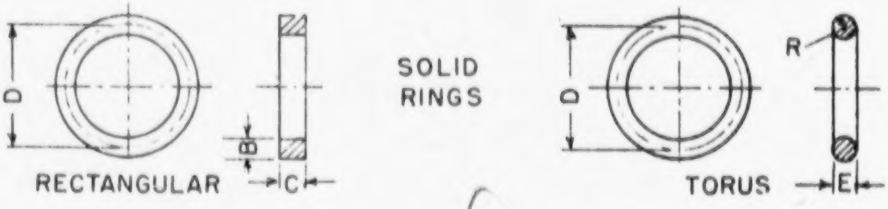
76. How much did the surface area of the ball increase after it was coated with the lead? __________

77. A water tank 6' in total length and 18" in dia. is in the form of a cylinder with two hemispherical ends. Find the area of sheet metal in the tank. __________

78. How many gallons will the tank hold? __________

79. Find the total weight of 150 1 1/4" dia. ball bearings made of steel. __________

80. What is the diameter of a sphere whose surface area is 154 sq. in. __________



81. Find the length of 3/4" round stock needed to form a ring 6" in dia. at the center line. ______________

82. Find the length of 1/2" round stock needed to form a ring 3.500" inside diameter. ______________

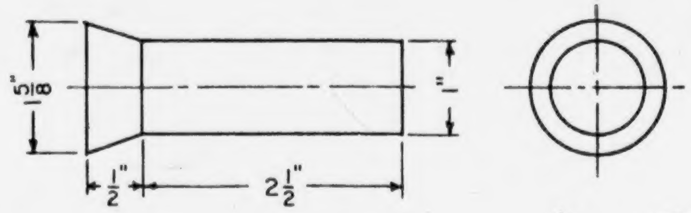
83. Find the outside diameter of a ring formed of 1 1/2" rod 17 1/2" long. ______________

84. The cross section of a solid wrought iron ring is a circle of 4" radius. The inner radius of the ring is 3'.

Find the area of the surface of the ring. ______________

Find the volume of the ring. ______________

85. Find the surface area of a ring with an outside dia. of 10" made of 1" round steel.



86. Find the volume of the cylindrical part of the rivet illustrated above. ____________

Find the volume of the head. ____________

The rivets are steel. Find the weight of 100. ____________

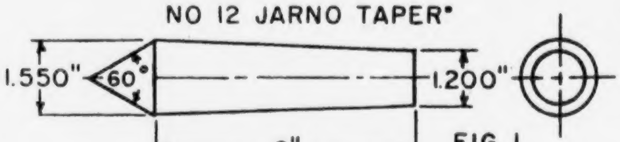


FIG. 1

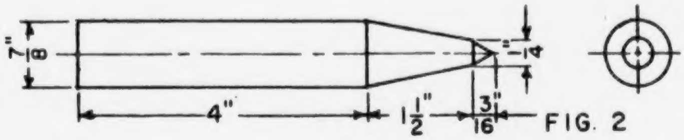


FIG. 2

87. Find the weight of the steel lathe center in Fig. 1.

____

88. Find the weight of 25 steel punches as in Fig. 2.

____

89. Find the number of cu. ft. of steel in 23 weights having the form of a frustrum of a square pyramid. Each is 3" high and 2" square on the bottom, and 1 1/2" square on the top. ____________

90. How many cu. in. os steel will there be in 11 tapered pins 8" long with diameters of 1 1/2" at the large end and 1" at the small end? ____________

91. How many cu. ft. of iron will there be in 5 castings made like Fig. 1? The hole is 3 1/2" dia. ____________

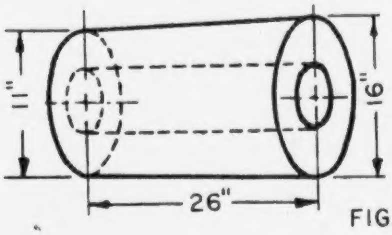


FIG. 1

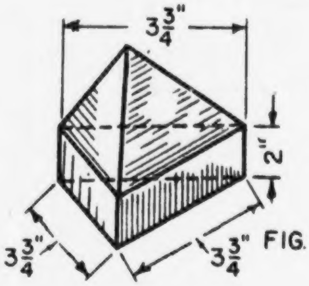


FIG. 2

92. What is the volume of the cast iron weight shown in Fig. 2? It is 5 1/2" high overall. ____________

93. Find the volume of a No. 7 Morse Standard plug. ____________

94. How many sq. Yds. of galvanized iron will be necessary to make the metal cover in Fig. 3. Make no allowance for seams How many cu. ft. will it displace? ____________ ____________

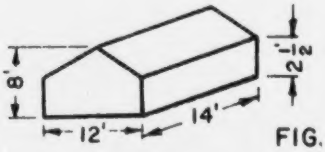


FIG. 3

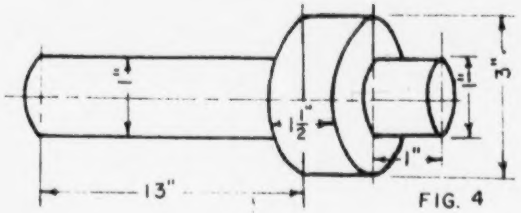


FIG. 4

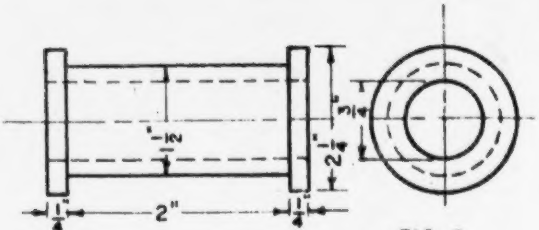


FIG. 6

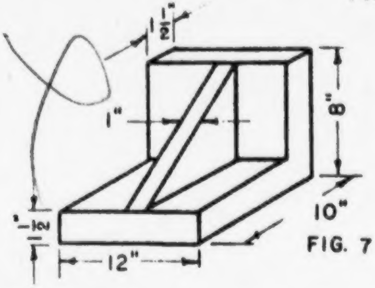


FIG. 7

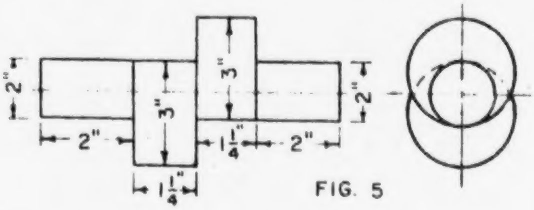


FIG. 5

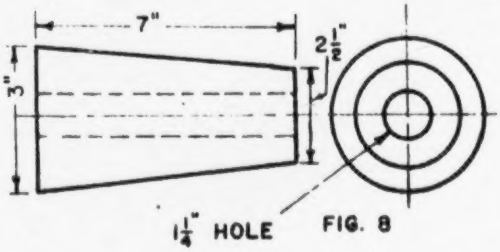


FIG. 8

95. A wrought iron washer 3 1/2" in dia. with a 1/2" hole is 5/32" thick. Find the number of these washers in a keg of 200 lbs.

____

96. What is the weight of 18 steel spindles as shown in Fig. 4?

____

97. What is the weight of the steel eccecntric in Fig. 5?

____

98. Find the weight of 30 cast iron spools as in Fig. 6.

____

99. Find the cost of the cast iron angle in Fig. 7 @ .22/ lb.

____

100. Estimate the cost of the bronze sleeve in Fig 8, if bronze costs 54¢/lb.

____

101. From a rectangular piece of sheet steel measuring 38" wide by 42" long and 1/2" thick, a circular plate 37" in dia. is to be cut. If this sheet metal weighs 10 1/4 lbs. per sq.ft., what will be the weight of the waste material?

____

102. A steel shaft weighing 844.46 lbs. has a length of 20'. What is its diameter? ________________

103. A steel plate (Fig. 9) has a base of 6" and a height of 4". If the plate is 3/8" thick, what does it weigh? ______________

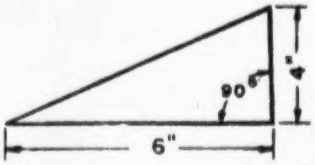


FIG. 9

104. What is the weight of the cast iron bench block as shown in Fig. 10? ______________

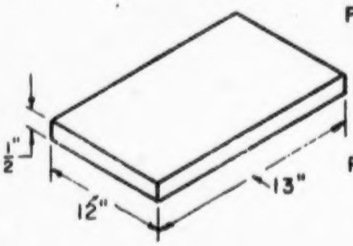


FIG. 10

105. Find the weight of the steel slide block as in Fig. 11. What is the cost @ 11 1/2¢ per lb. ______________

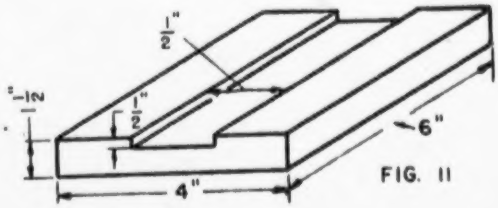


FIG. 11

106. Find the weight of a bar of madhine steel 4" in diameter and 5' long. ________________

107. Find the circumference, area in sq. in., volume, and weight, in pounds, of a 3/8" steel plate s 1/2" in dia.

____

108. A casting (cast iron) has a cored hole 4" in dia. and 20" long. How much will this hole reduce the weight of the casting?

109. 220 wrought iron blanks, 2" in dia. are to be shipped on a special order. The blanks have a 1" square hole through the center. The sheet material out of which they are to be made weighs 18 lbs./sq.ft. What will be the total weight of the 220 blanks?

110. Find the weight of the steel shaft, Fig. 12.

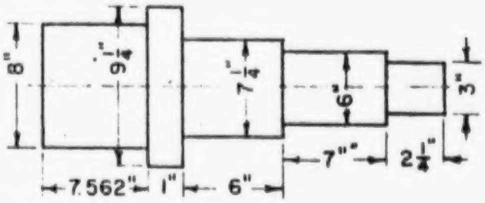


FIG. 12

111. What is the weight of the steel shoulder bolt, as in Fig. 13?

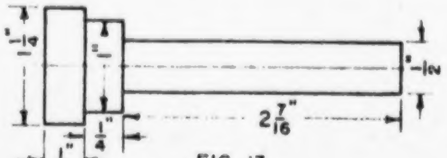


FIG. 13

112. Estimate the weight of the cast iron hand wheel as in Fig. 14

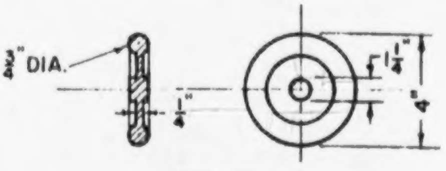


FIG. 14

113. Find the weight of the brass casting, Fig 15.

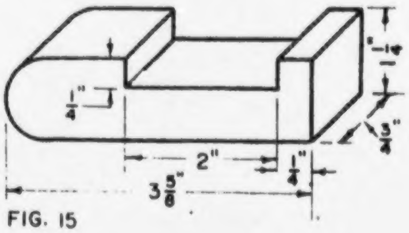


FIG. 15

114. How many cu. ft. of space are available in the metal cover Fig. 16? The peak is 7 1/2" high and the dimensions are the sides of two squares. ___________

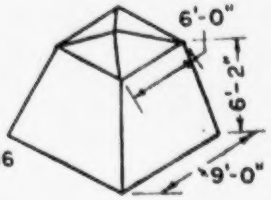


FIG 16

115. Find the volume, weight, and total area to be finished of this cast iron spacing block. Fig. 17 ___________

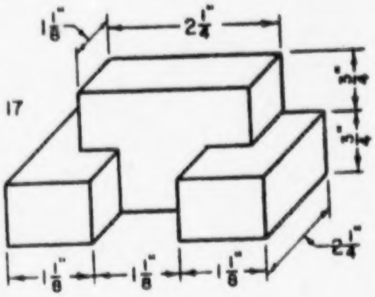


FIG. 17

116. Find the volume of this core box, Fig. 18. ___________

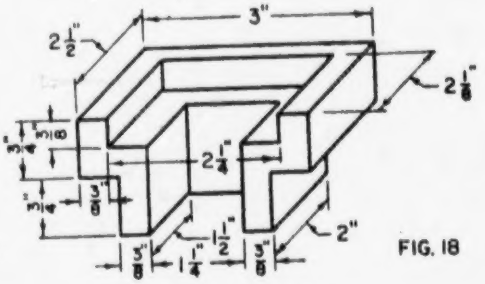
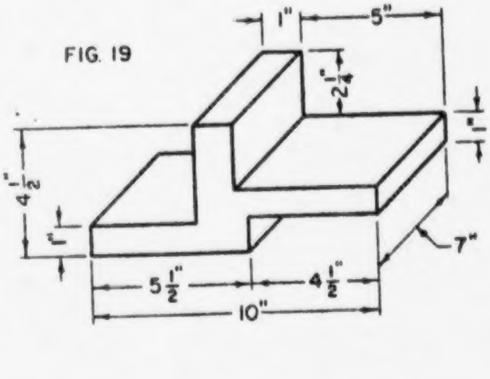


FIG. 18

117. Splice plate, cast iron. Fig. 19. Price is 3 1/4¢/lb. Find the cost of 800 lbs.

____

FIG. 19



118. Fig. 20, slide valve, cast iron. Find the volume and weight of one piece. ___________

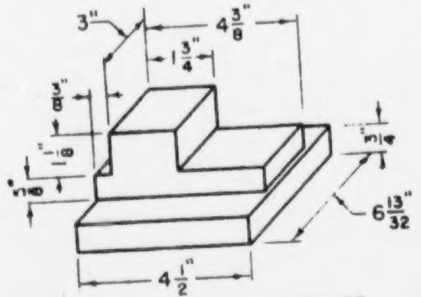


FIG. 20

119. Fig 21, Bronze, finished all over. Find weight of rough casting and finished bushing.

____

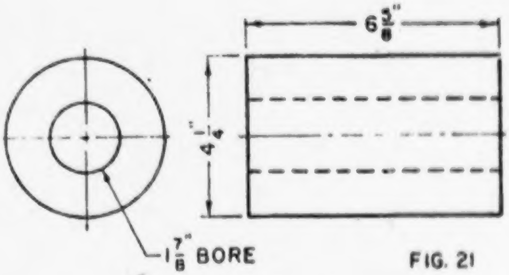


FIG. 21

120. Fig. 22, cast iron, price of castings is 4.8¢/ lb. Find the cost of 1000 castings.

____

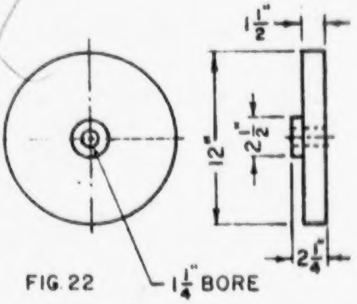


FIG. 22

121. Fig 23., two step gear, cast iron. Allow 1/8" for finish of all surfaces marked "f". Find the weight and cost of 1000 castings @ 9 1/4¢ per lb.

____

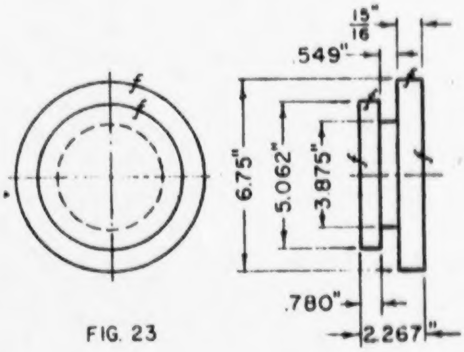


FIG. 23

122. Drop forging Fig. 24, steel. Find the weight and volume of one forging.

____

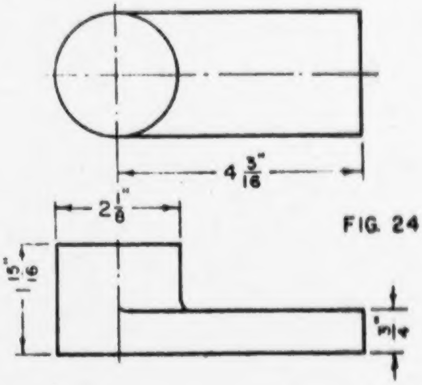


FIG. 24

123. Fig 25. Cast iron. Allow 1/8" finish on all suffaces marked "f". Use 1 3/8" core in hole. Price is 2 3/8¢/ lb. Find the cost of 250 castings.

____

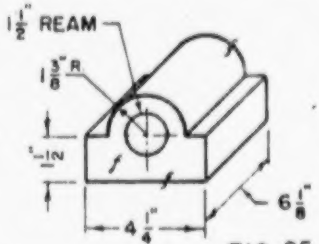


FIG. 25

124. Fig 26, steel. Find the volume and weight of one finished piece. ______________________

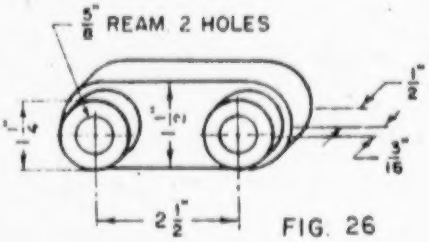


FIG. 26

125. Fig. 27, steel. Find the volume and weight of one finished piece. ______________________

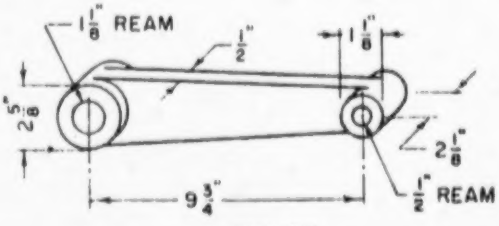


FIG. 27

126. Fig. 28, bevel pinion. Find the volume.______________________

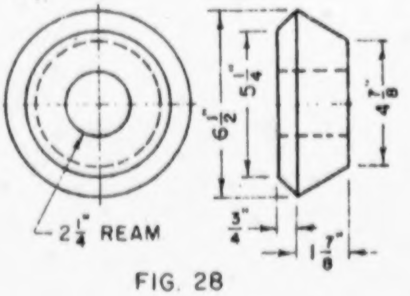


FIG. 28

Post evaluation:

The teacher will pick three problems for you to do from the work sheets you have just completed. One or two of the dimensions will be changed. There will be a time limit to this test.

S.I.C.S. Pak

SERIES: Preparation
GRADE: 12
NUMBER: Two
CLUSTER: Communication and Media
AREA: Broadcasting
Audio/Visual (Recording)
TITLE: Off and Recording -- the Wollensak

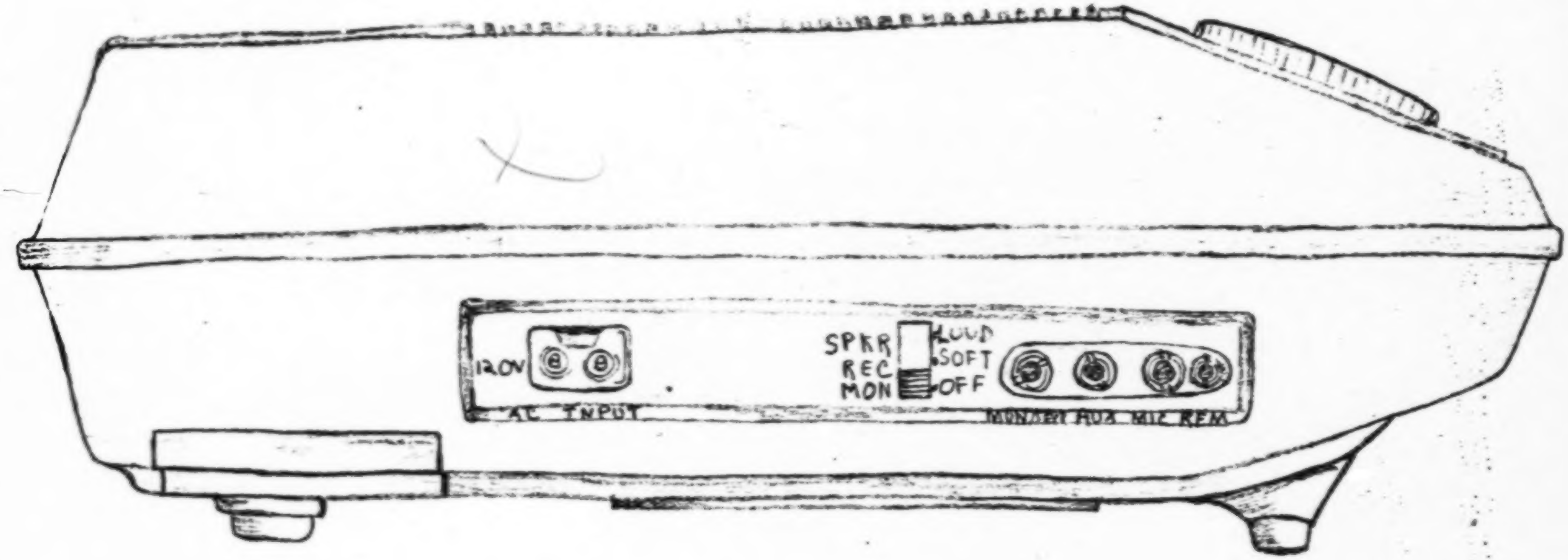
RATIONALE

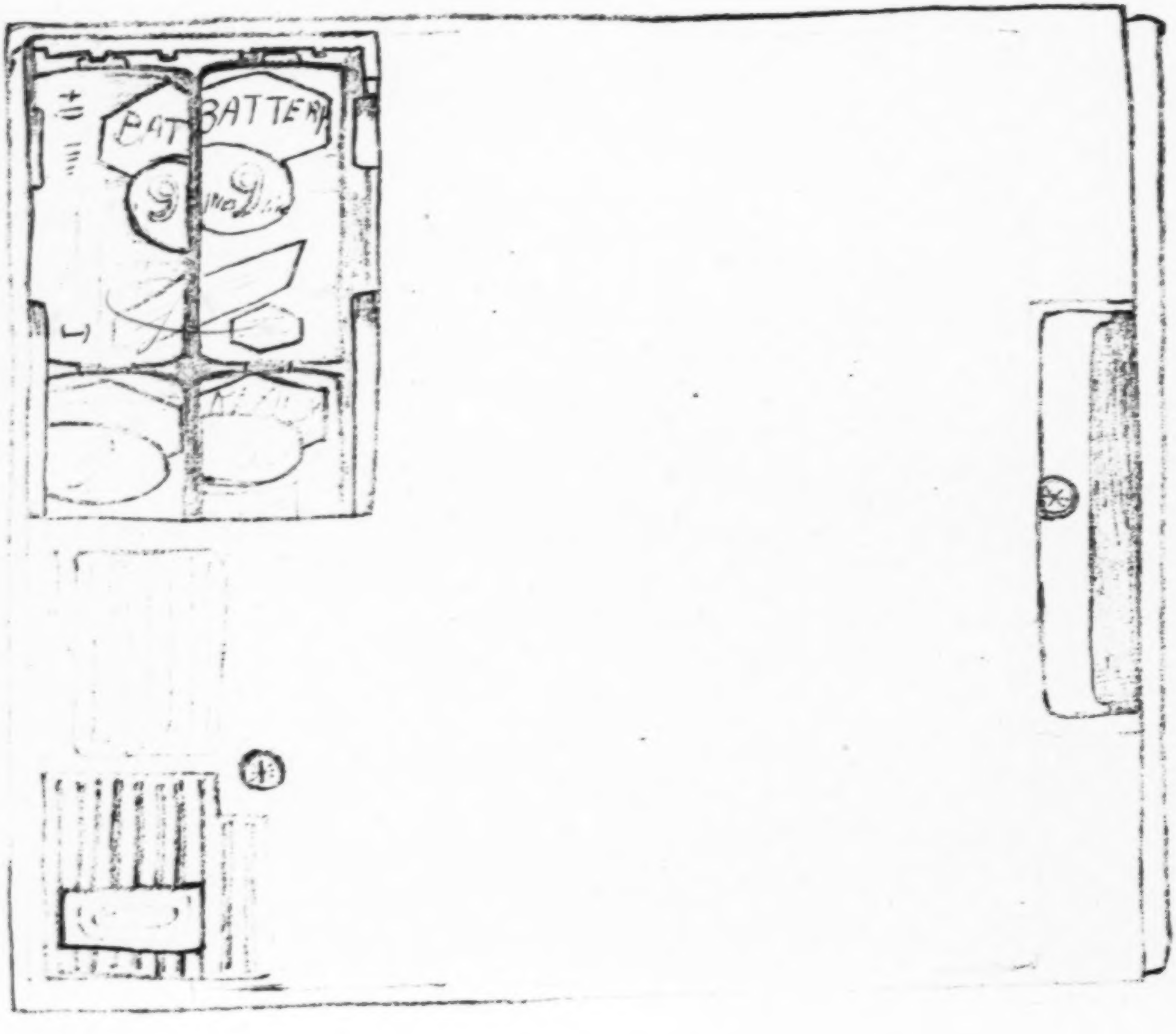
Before you can produce an interesting, well-done tape recording, you must first familiarize yourself with recording equipment.

You don't want to spoil your production because you forget to adjust the record level or have feedback noise.

On the other hand if you know your equipment, as a skilled professional must know his equipment, and feel comfortable and at ease operating it, chances are your final production will reflect the professional touch.

This package will teach you these important skills when you are operating the Wollensak 1520 tape recorder.


120V
AC INPUT
SPKR
REC
MON
LOUD
SOFT
OFF
MONITOR
AUX
MIC
REM


BAT
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CLUSTER: TRANSPORTATION

AREA: HIGHWAY TRANSPORTATION

TITLE: FRIENDS, ROMANS, COUNTRYMEN, LEND ME YOUR GEARS

BY: JIM MADEHEIM
SAHUARITA HIGH SCHOOL
MATH DEPARTMENT

Rationale:

To understand the operation of transmission, differentials, and fan belt systems, it is necessary to be familiar with ratios and proportions. This package demonstrates the use of ratios and proportions in the automobile.

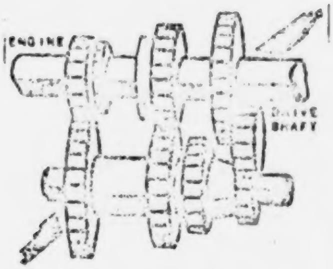
Behavioral Objectives:

1. Given the number of teeth or the dimensions of a set of gears, you will be able to find the ratios of all the gears to each other with at least 75% accuracy.

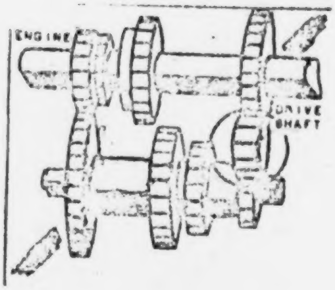
2. Given the necessary information to set up a proportion between gears or pulleys, you will be able to find the unknown quantity of the proportion with at least 75% accuracy.

Pre-Evaluation:

①



②



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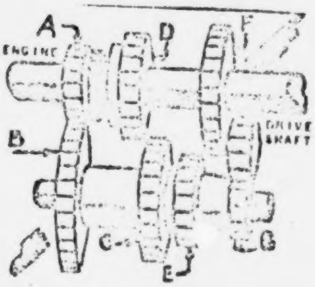

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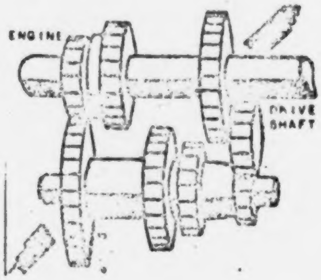
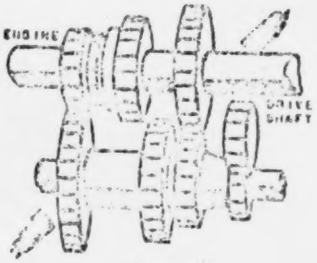


⑤

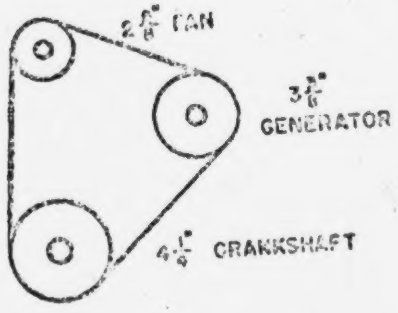


A = 14 TEETH
B = 28 TEETH
C = 21 TEETH
D = 21 TEETH
E = 16 TEETH
F = 26 TEETH
G = 12 TEETH

The five pictures above represent the five possible positions of a 3 speed transmission; first, second, third, reverse, and neutral, though not necessarily in that order.

1. Which transmission position is represented by each picture?

2. If A, B, C, and D are in mesh, what is the speed ratio of the transmission?



3. Find the r.p.m. of the fan pulley if the crankshaft is turning at 1500 r.p.m.

Data Brief # 1

Finding the ratio of one number to another is a method of comparing those two numbers by division. Thus, if a and b are two numbers and $b \neq 0$, the ratio of a to b is the number a/b. Keep in mind that you do not find the ration of one object to another. You find the ratio of two numbers which are measures of the objects. A ratio is anumber! In forming a ratio, the first number mentioned is put in the numerator. Examples:

If line l has a measure of 15" and line m has a measure of 20", the ratio of l to m is 15/20 = 3/4.

If A has a measure of 20° and B is a right angle, the ratio of A to B is 20/90, which is 2/9.

The ratio of 5" to 10' is 5/120 or 1/24.

You have to be working with the same units to find a ratio. All ratios must be expressed in lowest terms.

Data Brief # 2

A proportion is a statement that two ratios are equal. For example

$$\frac{a}{b} = \frac{c}{d}$$

is a proportion.

It tells you that the ratio of a to b and the ratio of c to d are the same number. This proportion can be read either "a is to b as c is to d" or "a divided by b equals c divided by d".

A very useful means of solving a proportion for an unknown term is "cross multiplication." Cross Multiplication works across an equal sign, not across a plus or minus sign. Examples:

$$\frac{a}{5} = \frac{4}{10} \qquad 10a = 4 \cdot 5 = 20$$

$$a = 2$$

$$\frac{b}{6} = \frac{7}{9} \qquad 9b = 42$$

$$b = \frac{42}{9} = \frac{14}{3}$$

In the above problem, it would be simpler to multiply each side of the equal sign by three first. This would make the problem simpler.

$$3 \cdot \frac{b}{6} = \frac{7}{9} \cdot 3 \qquad \frac{b}{2} = \frac{7}{3} \qquad 3b = 14$$

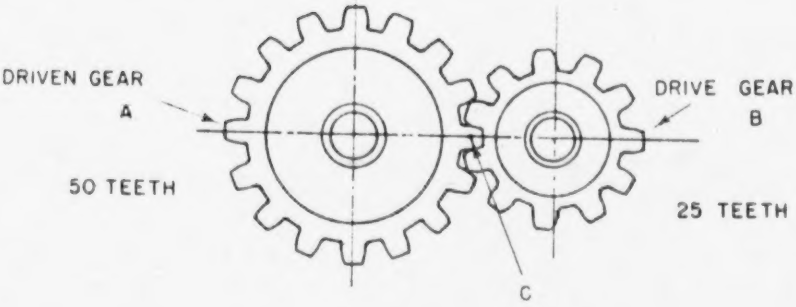
$$b = \frac{14}{3}$$

In the problem below, the first step would be to multiply each side of the equal sign by 1/4, and then cross multiply.

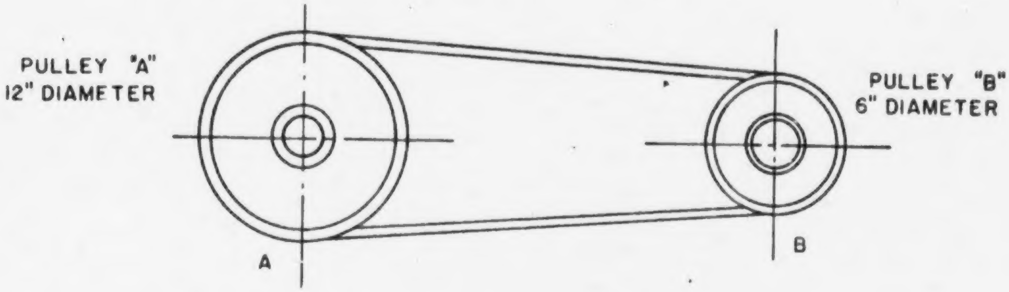
$$\left(\frac{1}{4}\right)\frac{64}{x} = \frac{36}{15}\left(\frac{1}{4}\right) \qquad \frac{16}{x} = \frac{9}{15} \qquad \frac{16}{x} = \frac{3}{5}$$

$$3x = 80 \qquad x = \frac{80}{3}$$

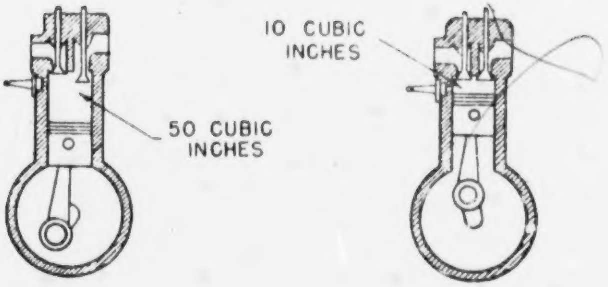
Data Brief # 3



In the above case, if gear A makes one turn, 50 teeth pass point C. Since the teeth in gear A are in mesh with the teeth in gear B the same number of teeth on gear B must pass point C. In order for 50 teeth on gear B to pass point C, gear B must make 2 turns. Therefore, gear B is turning twice as fast as gear A and the speed ratio of gear B to A would be 2:1. Spedd ratios are usually driver to driven.

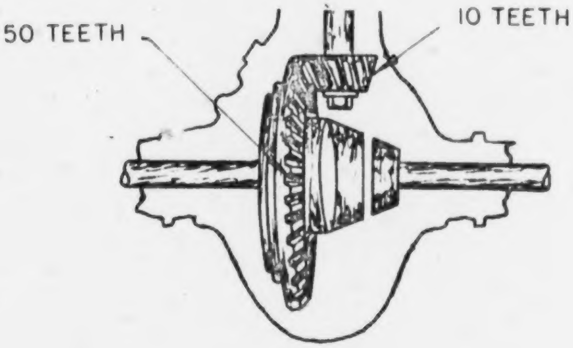


Pulley A is twice as large as pulley B. In one turn of pulley A, a length of belt equal to its circumference will move past any given point. In order for the same length of belt to cause motion in pulley B, pulley B must make 2 turns, (assuming no slippage). The speed ratio of pulley B to pulley A is 2:1.



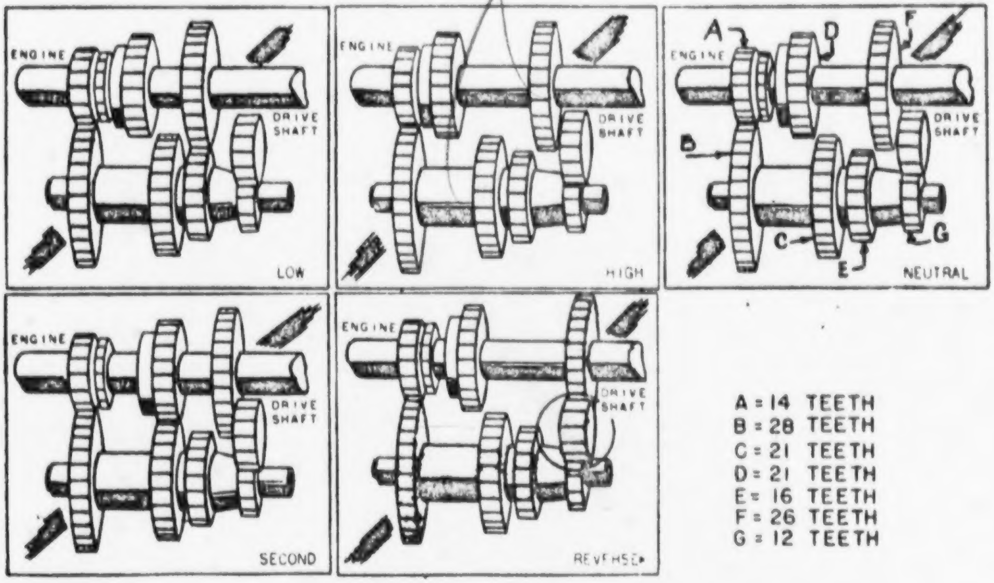
Compression ratio is a comparison between the amount of space (cubic inches) in the cylinder when the piston is at the bottom of the stroke, and the space when the piston is at the top of the stroke. If there was five times as much space when the piston was at the bottom as there was when the piston was at the top, the compression ratio would be 5:1.

Transmission ratio refers to the number of times the incoming speed is reduced by the transmission. A transmission ratio of 5:1 means the incoming speed (from the engine) is reduced 5 times by the transmission or that the rpm of the crankshaft is 5 times the rpm of the drive shaft.



Rear axle ratio (pictured above) refers to the number of times the speed is reduced by the ring gear and pinion.

To understand the pictures below better, use the picture on the preceeding page also. Note that gears D and F always turn with the output because of the splined shaft. All the gears on the layshaft (counter shaft) rotate together at the same speed



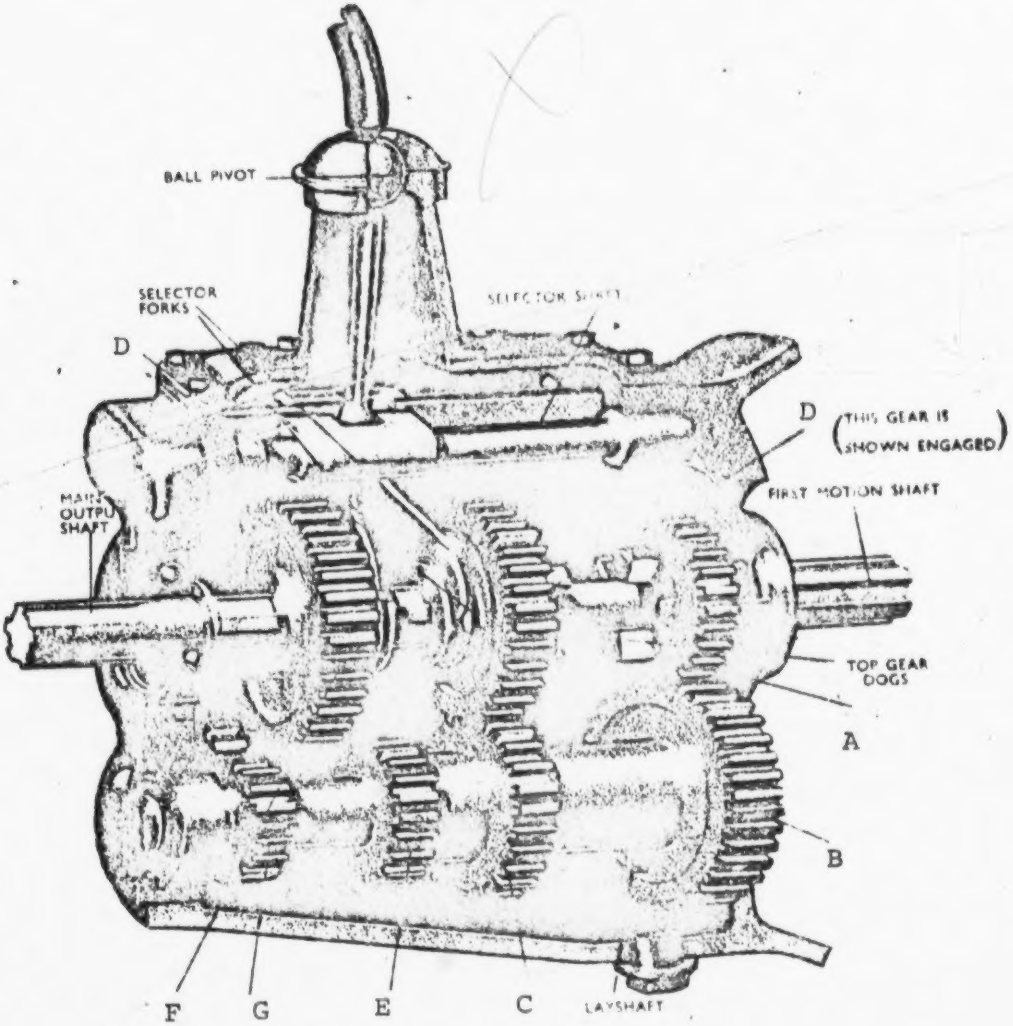
If the engine turns 728 times, gear A turns 728 times, gear B turns 364 times and so does gear E. Thus gear F turns 224 times. (The arithmetic is below).

$$\frac{A_{\text{teeth}}}{B_{\text{teeth}}} = \frac{14}{28}, \quad \frac{14}{28} = \frac{x}{728}, \quad x = 364$$

$$\frac{E_{\text{teeth}}}{F_{\text{teeth}}} = \frac{16}{26}, \quad \frac{16}{26} = \frac{x}{364}, \quad x = 224$$

A rear axle ratio of 5:1 means that the rpm of the drive shaft is 5 times as great as the rpm of the rear axle.

Both the transmission and differential serve to reduce the original rpm of the crankshaft. Considering both, we get a measure of the total gear reduction. For example, a transmission ratio of 4 and a rear axle ratio of 5, yield a total gear reduction of 20. Thus the crankshaft would turn 20 times while the rear axle turned only once.



Activity # 1

Express the ratio of x to y in simplest form .

1. $x = 4$ and $y = 6$ $\qquad$ $x = 3a$ and $y = 5a$

$x = 10$ and $y = 26$ $\qquad$ $x = 5a^2$ and $y = 20a^2$

2. $\frac{2x}{3y} = \frac{4}{7}$ $\qquad$ $\frac{3}{5}x = 17y$

$\frac{y}{x} = \frac{7}{11}$ $\qquad$ $\frac{6y}{9x} = \frac{36}{57}$

Express inlowest terms the ratio of:

3. 1 foot to 1 yard

3 ounces to two pounds

4 miles to 125 feet

a right angle to a straight angle

Express each of the following ratios in lowest terms, assuming no denominator is zero.

4. $\frac{2x - 6y}{x^2 - 9y^2}$ $\qquad$ 5. $\frac{6x^2 - 7x - 3}{2x^2 + x - 6}$ $\qquad$ 6. $\frac{x^3 - 8}{2x - 4}$

Find the ratio of x to y given that:

7. $\frac{x-y}{x+y} = \frac{2}{3}$

8. $\frac{1}{x-2y} = \frac{3}{x+y}$

9. $2x - y = 7$

$x + 2y = 11$

10. $\frac{x-4}{4} = \frac{y-3}{3}$

Activity # 2

Complete each statement:

1. If $\frac{a}{b} = \frac{2}{3}$ then $\frac{a+b}{b}$ = __?__ and $\frac{a-b}{b}$ = __?__

2. If $\frac{a}{2} = \frac{b}{3}$ then $\frac{b+3}{3}$ = __?__ and $\frac{b-3}{3}$ = __?__

3. If $\frac{a+b}{b} = \frac{10}{3}$ then $\frac{a}{b}$ = __?__ and $\frac{a-b}{b}$ = __?__

4. If 3a = 4b, then $\frac{a}{b} = \frac{?}{?}$ and $\frac{?}{b} = \frac{?}{a}$

Find the value of x in each proportion.

5. $\frac{x+1}{4} = \frac{2}{3}$

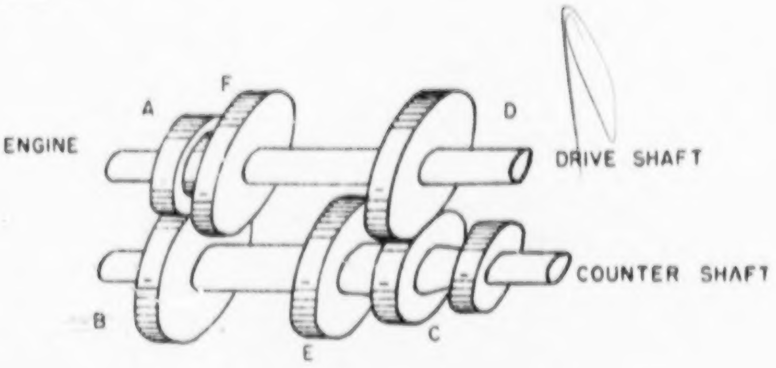
6. $\frac{x-2}{3} = \frac{x}{4}$

7. $\frac{x}{4} = \frac{9}{x}$

8. $\frac{x-3}{1} = \frac{7}{x+3}$

The following material has been deleted: Activity #3.

POST-EVALUATION:



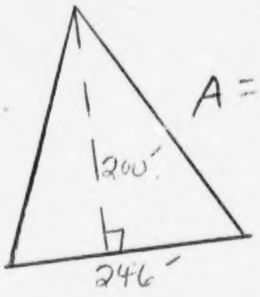
1. In the transmission pictured above, gear A = 16 teeth, gear B = 32 teeth, gear C = 18 teeth, gear D = 24 teeth. The crankshaft is turning 2100 rpm. What is the rpm of the drive shaft? ____________

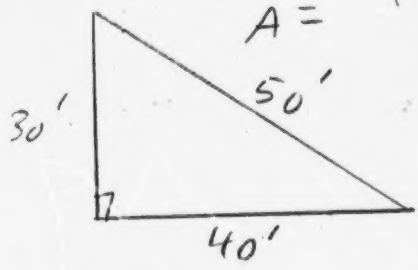
2. In second speed in this transmission. gear E = 27 teeth meshes with gear F (24 teeth). What is the rpm of the drive shaft if the crankshaft is turning 2000 rpm. ____________

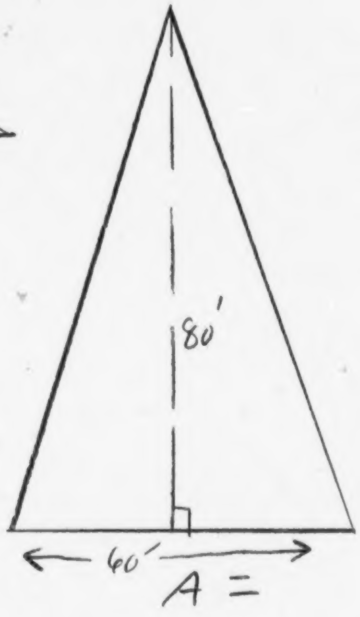
3. Take the pre-evaluation test.

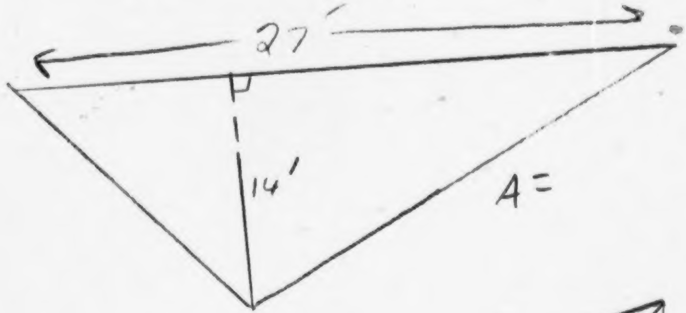
Activity # 4

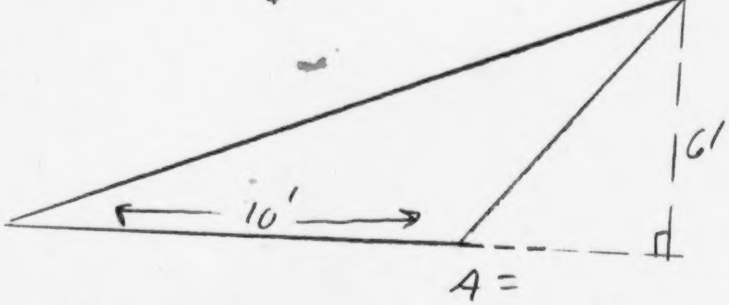
Find the area of each triangle below.

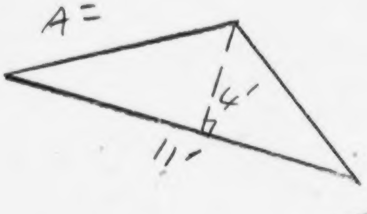


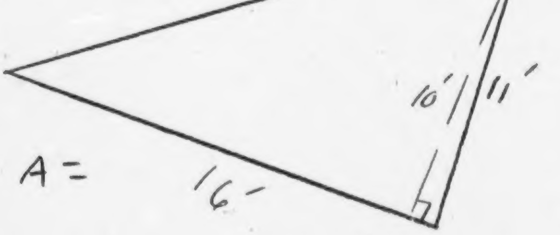


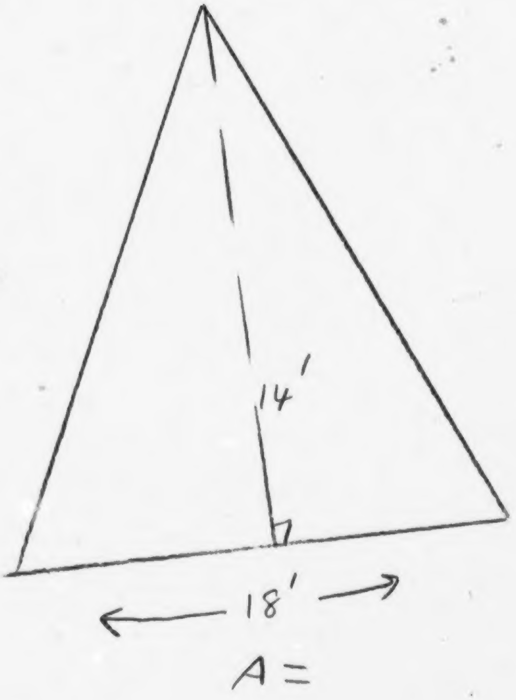












A man contracts to paint the gables on 17 houses ina new housing development. (Picture below is a gable).

What is the area of one gable? ____________

What is the total area for 17 houses to be painted? (There are two gables on every house) ____________

To insure a good job, he will use a latex paint which costs $5.49/gall. Since the gables are smooth wood, he will be able to cover 400-450 sq.ft. with one gallon.

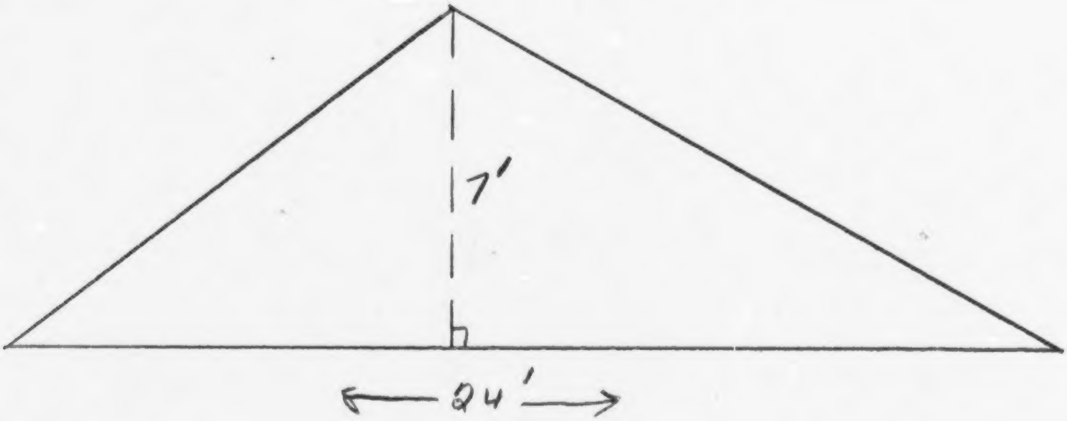
How many gallons will he use? ____________

What is the cost of the paint? ____________

He bought the paint in a discount store. Ina regular paint store it would cost $8.95/gall.

How much would the paint cost at this higher price? ____________

How much did he save? ____________



A painter has to buy paint to do 6 houses all built by the same person. On one side of each house there is only one big round window, (see picture). It is 6' in diameter. What is the total area to

What is the total area, including the window for one side of these houses?

What is the radius of the window?

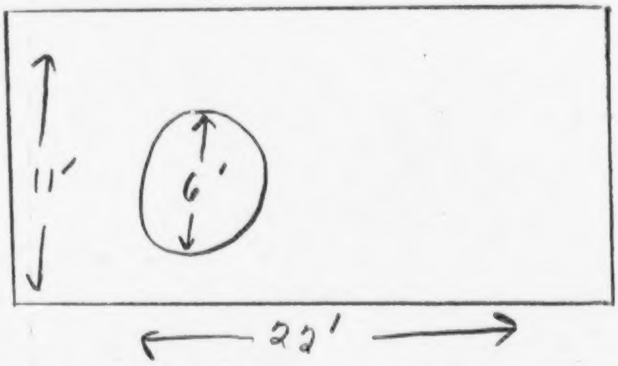
What is the area of the window?

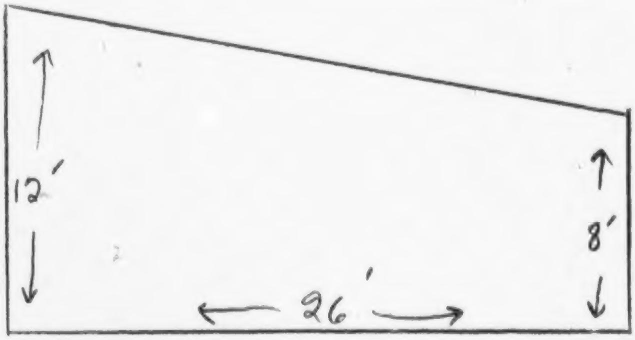
What is the area to be painted for one house?

What is the area to be painted for the 6 houses?

Masonry paint will be used, which covers up to 45o'/gall. How many gallons will be used?

A gallon costs $3.98 and a quart is $1.45. What is the cost for the paint job?





A house is built with two sides as pictured above on the right. One side has no windows because it faces west. The other side has one window 4' by 8'.

What is the area of the windowless side?

What is the area of the window/

What is the area of the side with the window that gets painted?

What is the total area to be painted?

The same paint as in the previous problem will be used. How much paint is bought?

What is the cost?

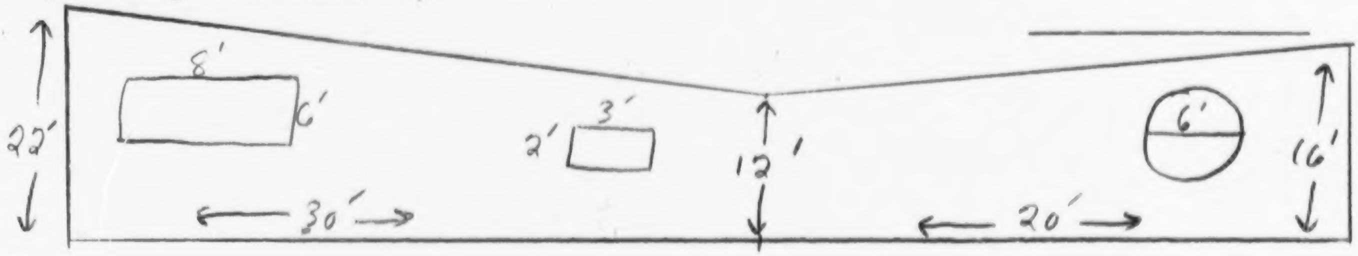
Post test:

The picture below is on the side of a building that is to be painted

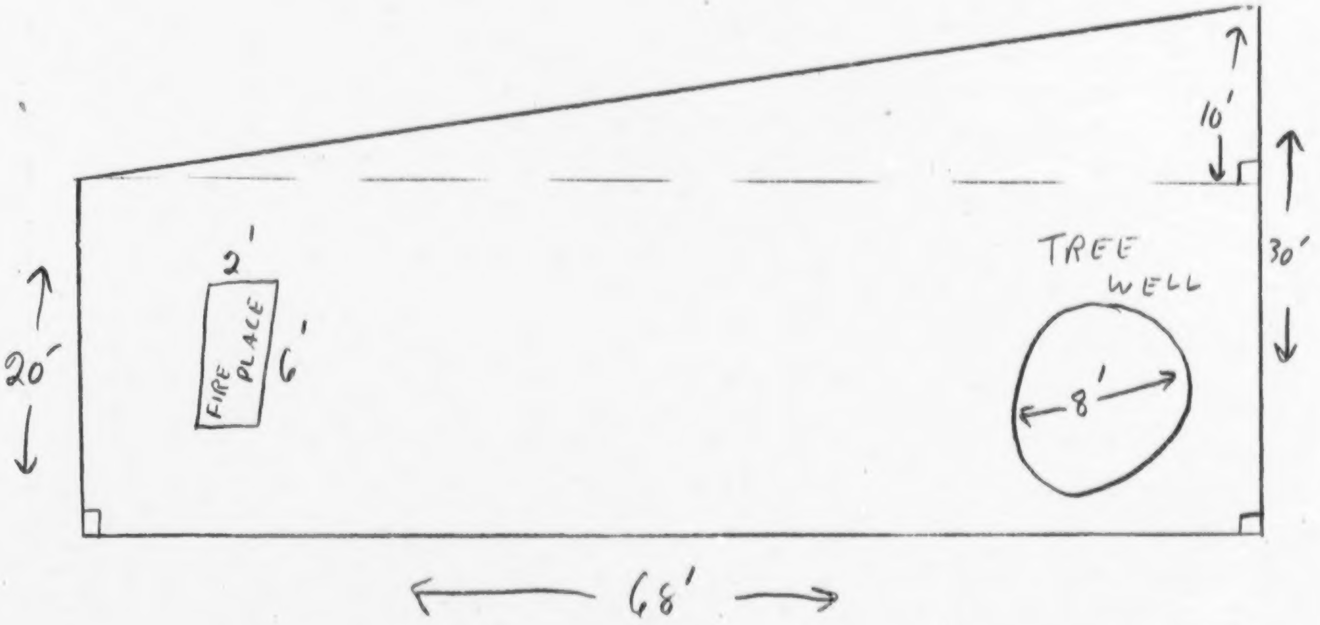
What is the area to be painted? ________

How many gallons of paint are needed? ________
(one gall. covers 300-400 sq.ft.)

The cost of the paint is $3.12/gall. What is the cost of painting this building? ________



The picture below is of a backyard patio to be covered with turf. Find the total area to be covered. (The fireplace and tree well do not get covered). ________



S. I. C. S. PAK

(STUDENT'S INDIVIDUALIZED CAREER SOURCE PACKAGE)

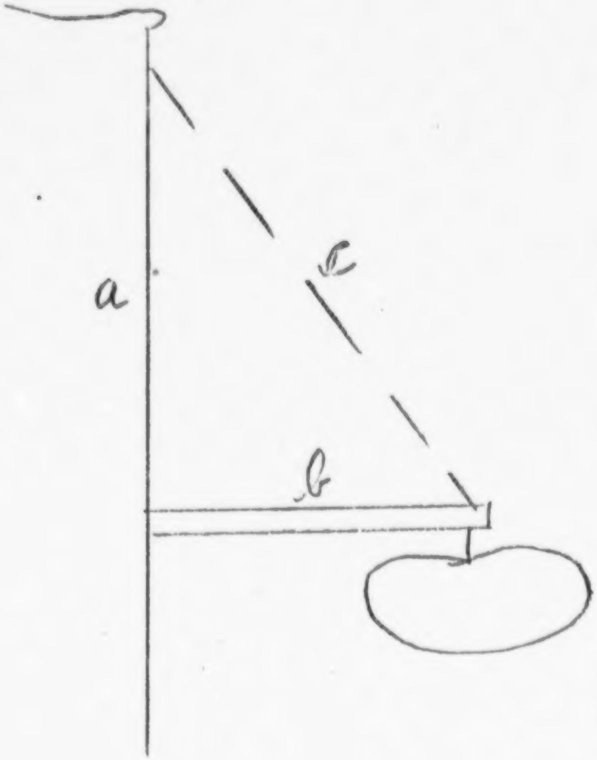
SERIES: Exploration

NUMBER: 370.3

CLUSTER: Marketing and Distribution

AREA: Advertising

TITLE: Hanging Up a Sign



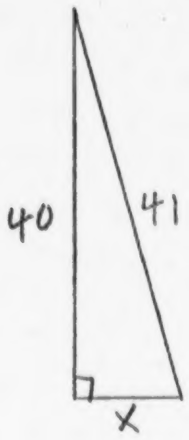
Referring to the picture above, find the length of the cablr(c) for each problem below.

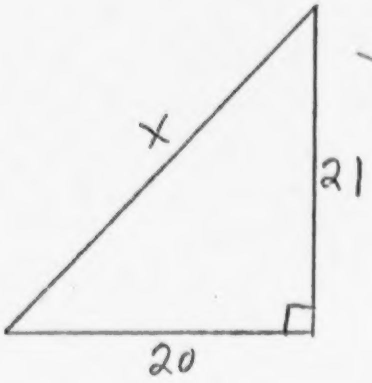
1. a = 22, b = 20

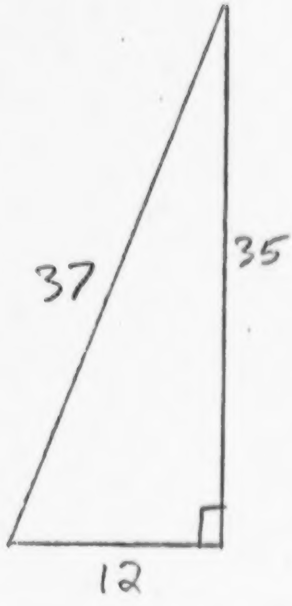
2. a = 42, b = 40

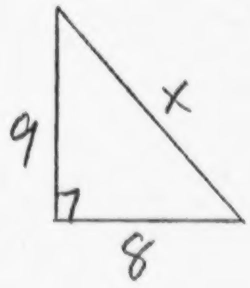
3. a = 17, b = 12

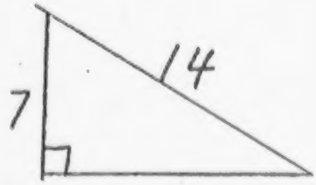
Post Test





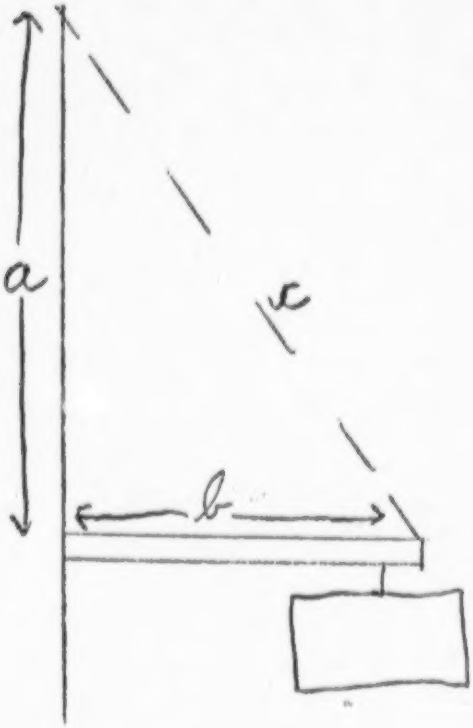






In each triangle above, find the length of the side x.

Activity # 4



Find the length of the cable (c) for each problem below.

1. a = 20, b = 12

2. a = 46, b = 10

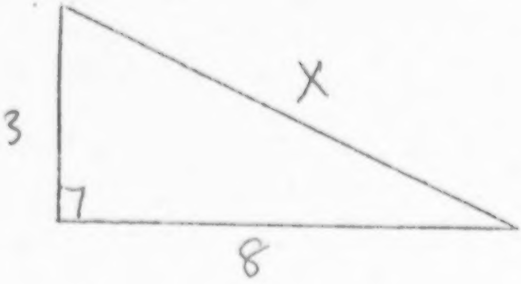
3. a = 16, b = 11

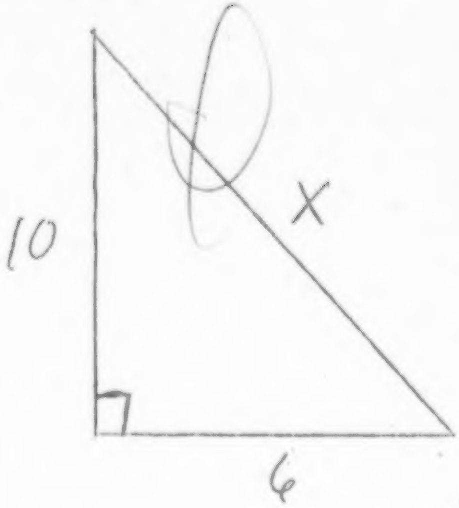
4. a = 24, b = 19

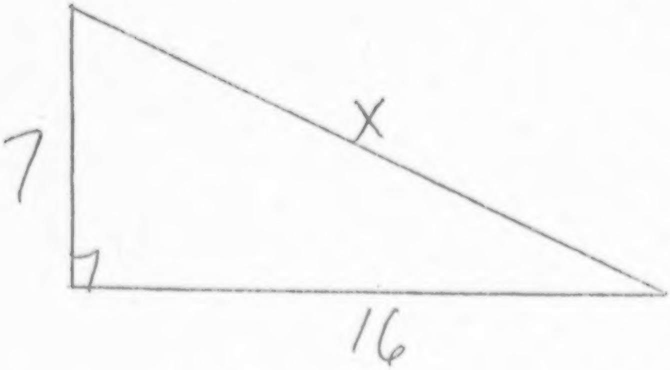
5. a = 60, b = 11

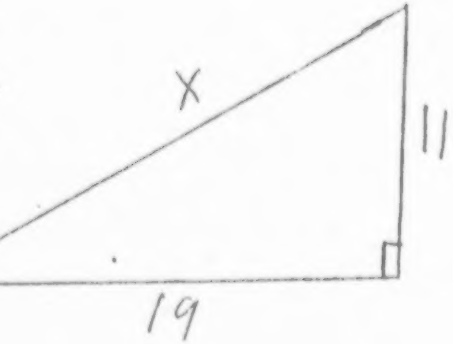
Activity # 4

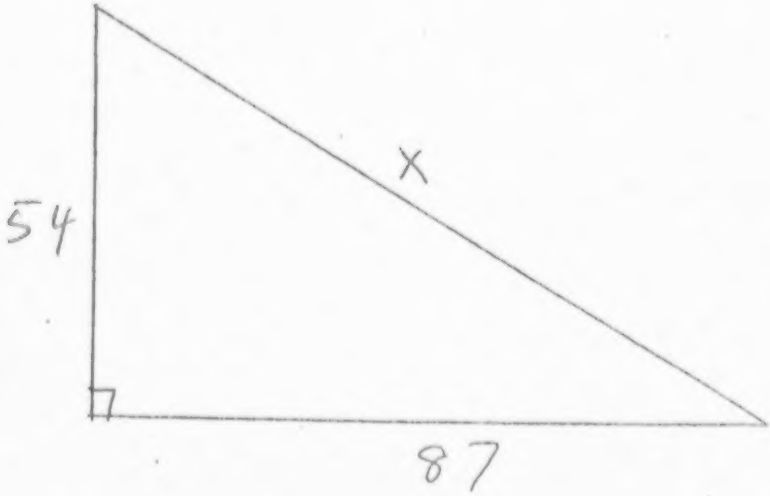
Find the third side of the triangle in each case











Activity # 4

Look up the following values in the book of tables.

$\sqrt{36}$

$\sqrt{144}$

$\sqrt{151}$

$\sqrt{324}$

$\sqrt{729}$

$\sqrt{961}$

$\sqrt{3}$

$\sqrt{15}$

$\sqrt{18}$

$\sqrt{22}$

46^2

57^2

58^2

99^2

567^2

824^2

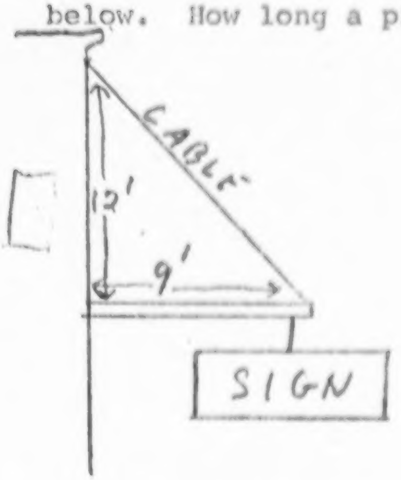
777^2

666^2

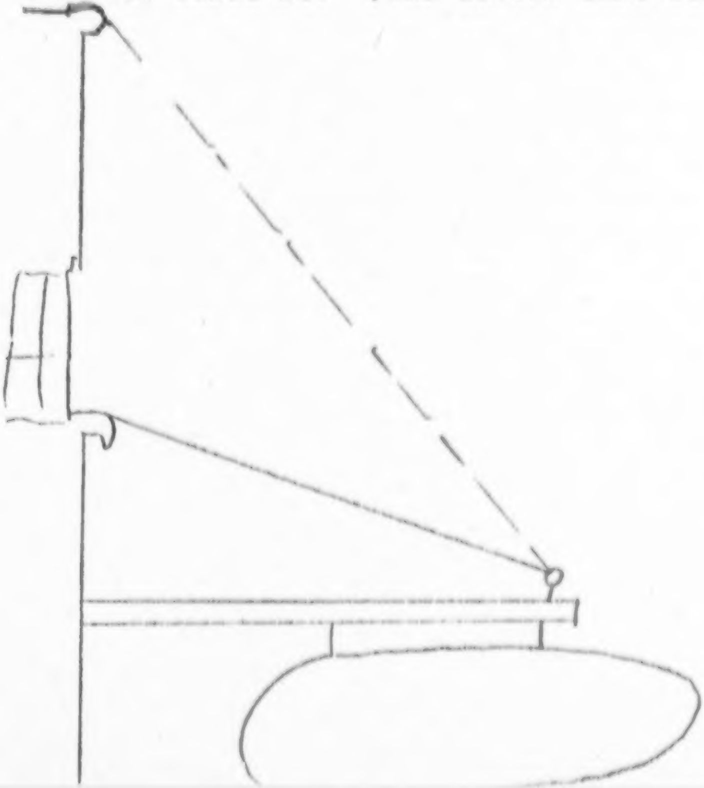
200^2

Activity # 3

A sign company has to hang a sign as in the picture below. How long a piece of cable should he make up?

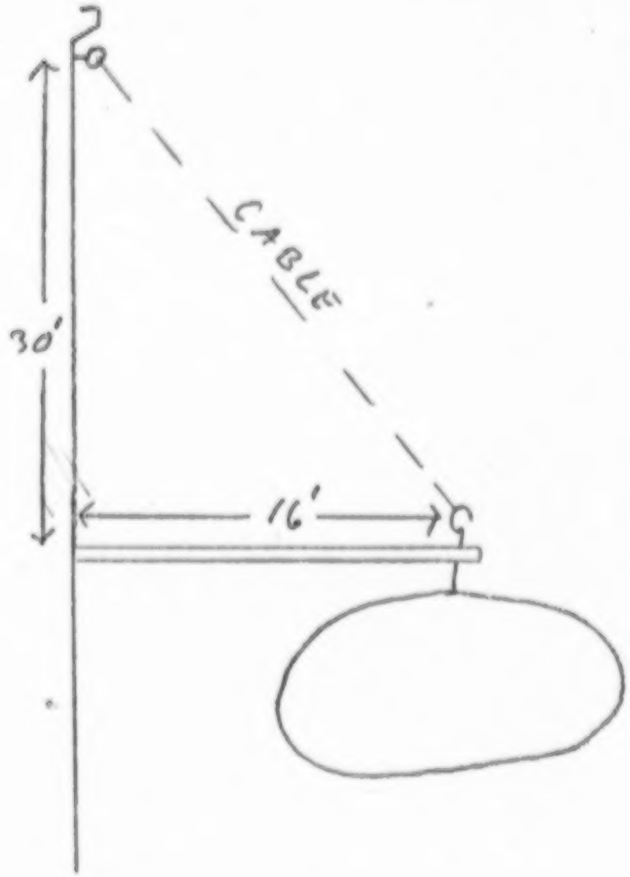


The cable (x) as in the picture below, helping to support the sign is cracking the window sill that it is attached to. A new cable will replace it and will be attached to the roof. The bar the sign is attached to is 16' below the roof and the end of the bar is 12' from the building. How long should the new cable be?)The dotted line sill be the new cable.)



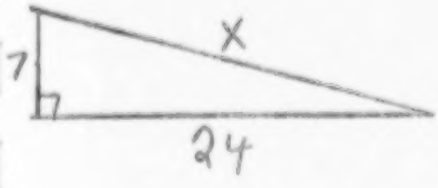
Activity # 3

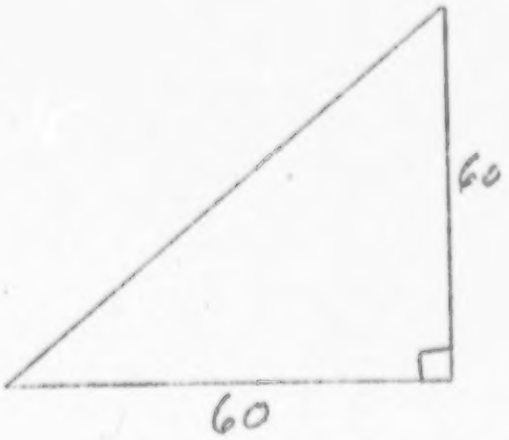
A sign is to be hung as pictured. The cable support on the building is 30' above the sign and the hook on the sign is 16' from the building. How long a cable is needed to help support this sign?

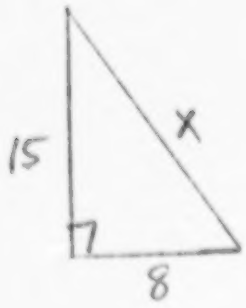


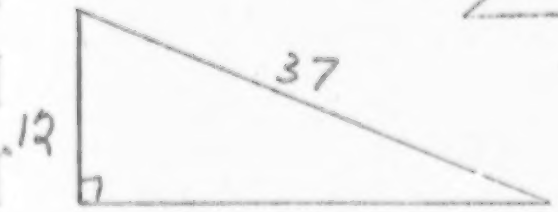
Activity # 3

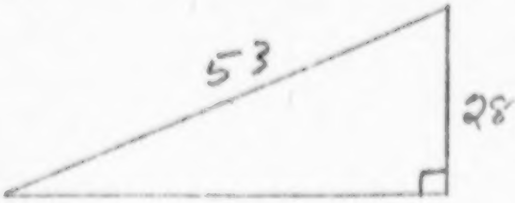
Find the third side of each right triangle. The answer in each case will be a whole number.

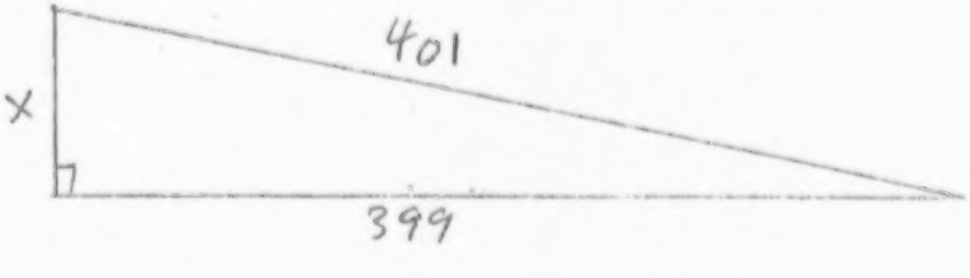


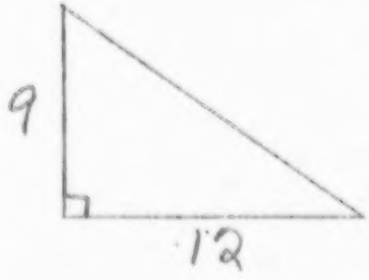


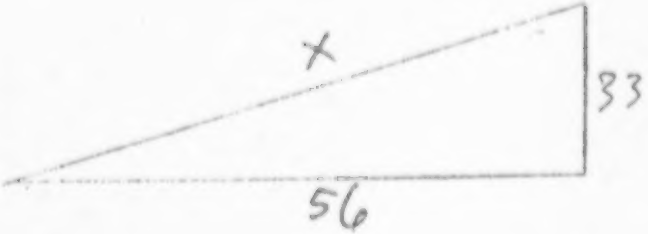












Activity # 2

In each pair of numbers below, the first is the length of the longest side of a right triangle and the second is the length of one of the shorter sides. Find the length of the third side.

5,4
26,24
20,16
5000,3000
35,28
1301,51
365,364
409,391
1229,1221

10,6
15,12
50,30
100,60
1409,159
269,260
229,221
221,220

15,9
10,8
500,300
2500,1500
421,420
449,351
281,231
221,171

Activity # 1

Each set of three numbers below represents the lengths of the three sides of a triangle. Indicate by writing "yes" or "no" after each triplet that, yes, the triangle is a right triangle, or no, the triangle is not a right triangle.

6,8,10
26,19,24
7,25,24
9, 41,40
583,1344,1465
901 1260,1549
1889,1360,1311
5,13,12
7,8,9
15,12,8
10,20,30
481,29,490
325,204,222
1044,1165,517
2,7,8
12,15,9
12,16,20
11,61,60
483,484,42
610,1189,1020

Data Brief # 4

You are going to need your sheet of square root tables at this time. It is attached to this package.

Look in the column titled " No. " and find "16". Look in the column titled "square root " directly across from the "16" and you should find the number "4". This means that the square root of 16 is 4; or written in math symbals

$$\sqrt{16} = 4$$

Find the square root of 3 ($\sqrt{3}$)

Do you see that

$$\sqrt{5} = 2.24$$

$$\sqrt{7} = 2.65$$

$$\sqrt{143} = 11.96$$

To find the length of the third side of the triangle below set up the problem in the following manner.

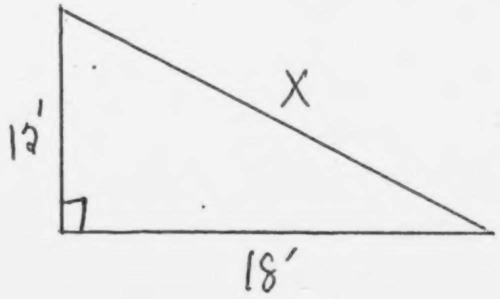
$$12^2 + 18^2 = x^2$$

$$144 + 324 = x^2$$

$$468 = x^2$$

$$\sqrt{468} = x$$

$$21.633 = x$$



Here is another examnle. If the longest side is 26', and one of the short sides is 24', how long is the other short side?

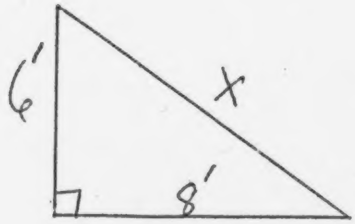
$26^2 = 24^2 + x^2$

$676 = 576 + x^2$

$676 - 576 = x^2$

$100 = x^2$

$10 = x$



Data Brief # 3

In the trigngle above you find the third side bv setting up the following equations and solving for the unknown.

$$6^2 + 8^2 = x^2$$

$$36 + 64 = x^2$$

$$100 = x^2$$

$$10 = x$$

In the triangle below you solve for the length of the unknown side in the following manner.

$$18^2 + x^2 = 30$$

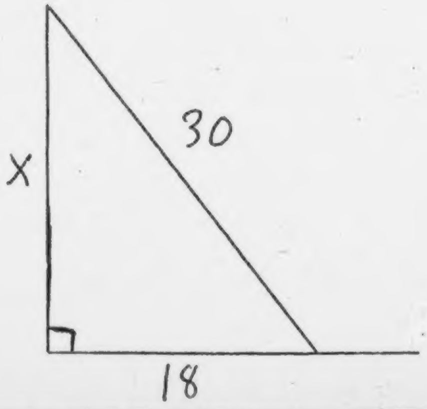
$$324 + x^2 = 900$$

$$x^2 = 900 - 324$$

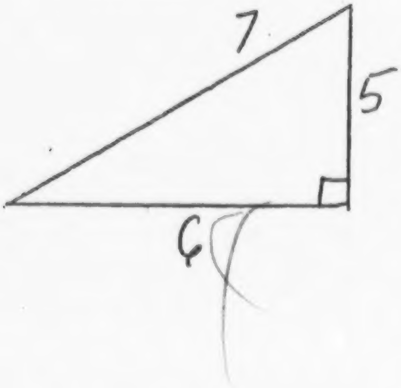
$$x^2 = 676$$

$$x = \sqrt{676}$$

$$x = 24$$



Can the triangle below be a right triangle?



$5^2 + 6^2 \overset{?}{=} 7^2$

$25 + 36 \overset{?}{=} 49$

$61 \neq 49$ Therefore it is not a right triangle.

Data Brief # 2

If the side "b" in the triangle below is 6' and the longest side "c" is 10', the length of the other short side is found by subtracting.

$a^2 + b^2 = c^2$

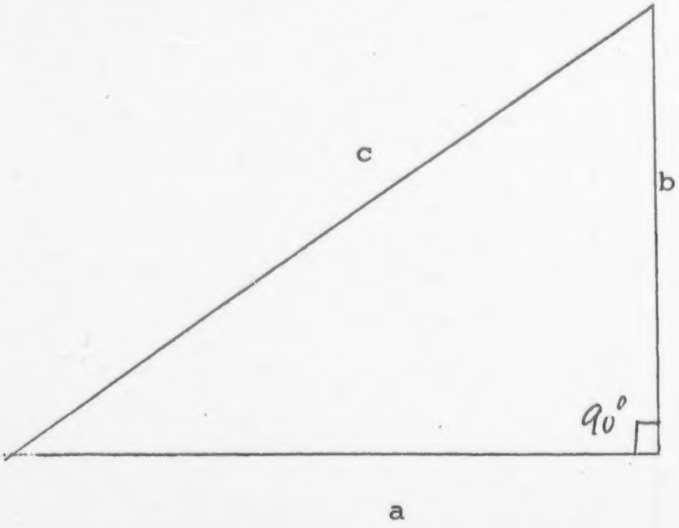
$a^2 + 6^2 = 10^2$

$a^2 + 36 = 100$

$a^2 = 100 - 36$

$a^2 = 64$

$a = 8$



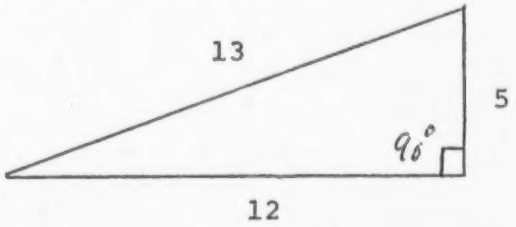
Information sources.

1. Read Data-brief #1
2. Read Data brief #2
3. Read Data Brief #3
4. Read Data Brief #4

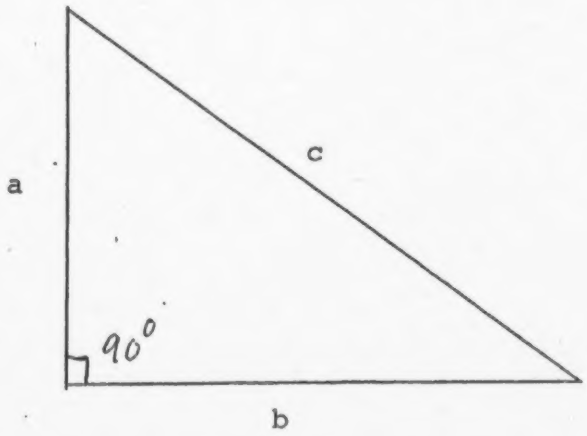
Data Brief #1

A right triangle has one angle that measures 90°.

In any right triangle, the length of one short side squared (a^2), plus the length of the other short side squared (b^2), equals the length of the longest side squared (c^2). $a^2 + b^2 = c^2$



$$5^2 + 12^2 = 13^2$$
$$25 + 144 = 169$$

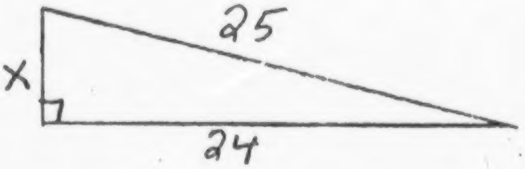


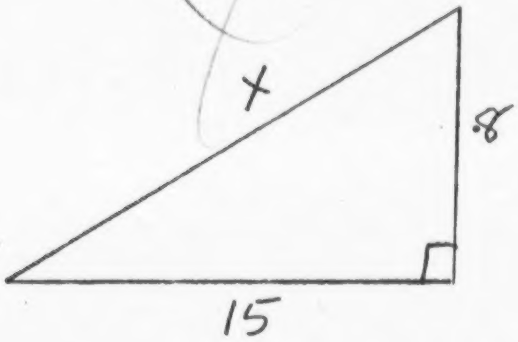
$a = 3, \quad b = 4, \quad c = ?$

$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = c^2$$
$$9 + 16 = c^2$$
$$25 = c^2$$
$$5 = c$$

Pre-Test:

Find the length of the third side of each triangle below.





On the

In the picture accompanying the objectives, if a = 24, b = 18, Find c.

Using the same picture, if a = 20, and b = 14, Find c.

Rationale.

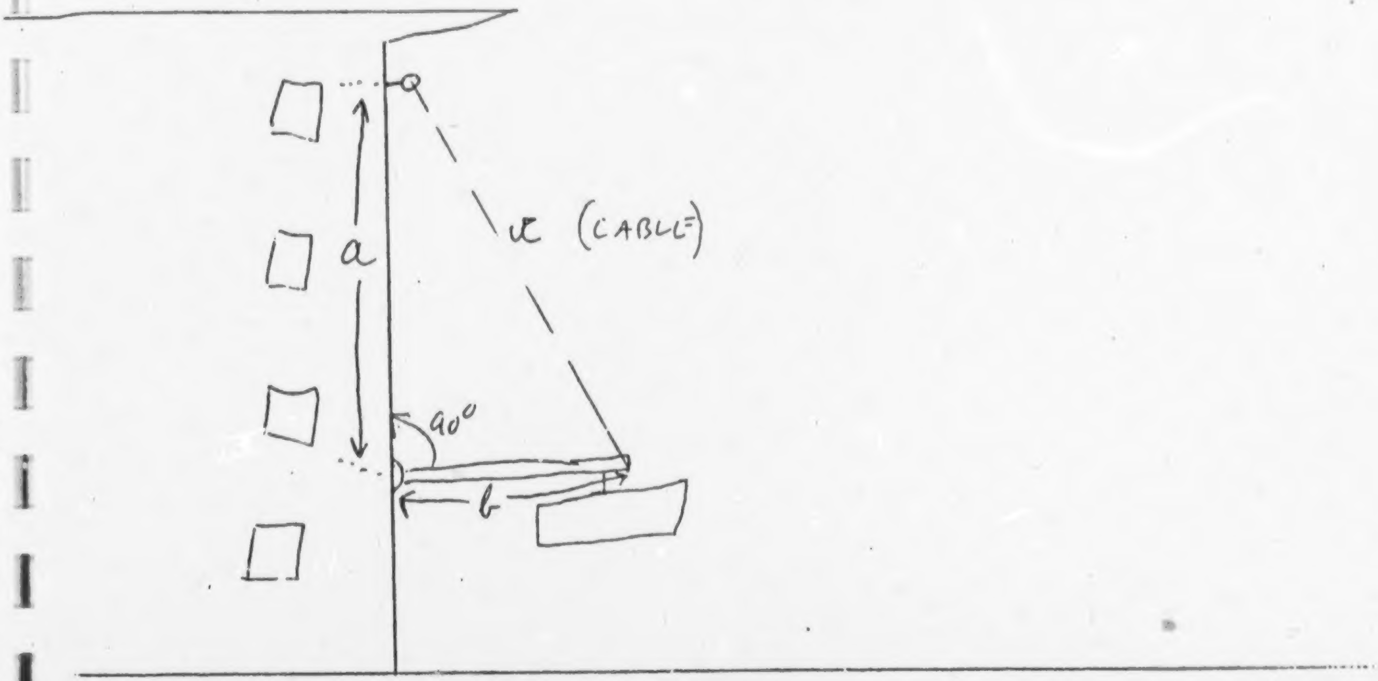
At times a distance cannot be measured directly. For example: How do you determine the length of the cable needed in the picture below before the sign is put up?

One method of finding the length of cable needed is by use of the Pythagorean theorem. This method of indirect measurement uses the fact that once you know two sides of a right triangle, you can determine the third side.

Behavioral Objectives:

Given any two sides of a right triangle, you will be a able to find the third side with two decimal place accuracy.

Given the distance "a" and the distance "b" as in the picture below, representing two legs of a right triangle, and the use of the electronic calculator and square root tables, you will be able to find the length of the cable with an accuracy of one foot.



S. I. C. S. PAK

(STUDENT'S INDIVIDUALIZED CAREER SOURCE PACKAGE)

SERIES: EXPLORATION

NUMBER: 372.4

CLUSTER: CONSTRUCTION

AREA: LAND DEVELOPMENT - SITE PREPARATION

TITLE: THROUGH A MOUNTAIN, ACROSS A CANYON

6

Rationale

One of the longer tunnels ever built goes through a mountain in France. Before the tunnel was started, the planners of the tunnel had to know how long it would be so that they could extimate its cost and appropriate the right amount of money. But you can't walk through a mountain so some other method of measuring had to be used. This package explains one such method.

Behavioral objectives

Given any angle between 0 and 90° you will be able to look up its tangent in a trig table with 100% accuracy.

Given the tangent of an angle, you will be able to name the nearest whole angle with 1°accuracy.

Given a distance to measure that requires the use of the tangent formula, you will be able to find the distance wanted to the nearest whole number using the tangent formula, trig tables, and calculator.

Pre-Test

Tan 63° =

Tan 71° =

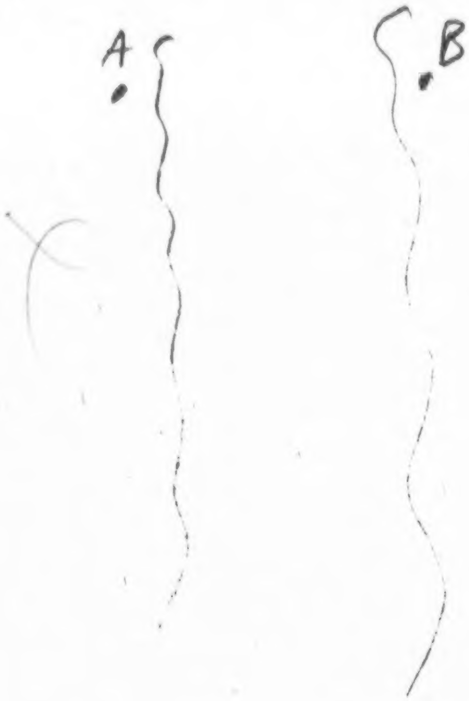
Tan 32° =

Tan x = 1.0000, x =

Tan x = .6605, x =

Tan x = .8201, x =

You need to find the distance from A to B. Draw a triangle that will allow you to use the tangent formula to find the distance AB. When you have drawn your triangle triangle, ask the teacher for the measurements you will need to know.



Information sources:

Read Data Brief # 1	"Using the tangent table"
Read Data Brief # 2	"Further use of the trig tables"
Read Data Brief # 3	"Common fractions and decimals"
Read Data Brief # 4	"Using the tangent formula"
Read Data Brief # 5	"Using the theodolite"

Data Brief # 1

Get a trig table from the teacher.

Look in the column headed "angle" and find 20°. Look across from 20° in the column headed "tangent" and you should see ".3640" This means the tangent of 20° is .3640 or in shorter form

Tan 20° = ,3640

Do you see that the tangent of 45° is 1?

Do you see that Tan 76° = 4.0108

Do you see that Tan 59° = 1.6643

What is the tangent of 42° ________________

You should have written down .9004.

Data Brief # 2

Question: The tangent of what angle is .8098?

Answer: 39° is the angle.

Question: Tan x = .5774 What is x

Answer: x = 30°, because Tan30° = .5774

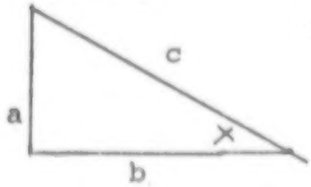
The symbol θ is a Greek letter called "theta". Many times greek letters instead of our own letters are used to designate anles.

Tan θ = 2.9042, θ = ______________

In the blank above you should have written 71°

Data Brief # 3

Definition: $$\tan x = \frac{\text{length of side opposite angle } x}{\text{length of side adjacent to angle } x}$$



In the triangle above $\tan x = \frac{a}{b}$

If a = 5' and b = 7' then $\tan x = \frac{5}{7} = ,7143$

Look in your trig tables and find the number in the tangent column closest to .7143.

Do you see that .7265 is closest?

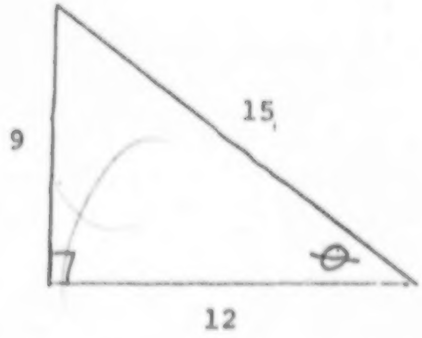
This means that angle x = 36° (approximately).

Find $\tan x = 7/8$, $x =$ _______________

$\tan x = 15/11$ $x =$ _______________

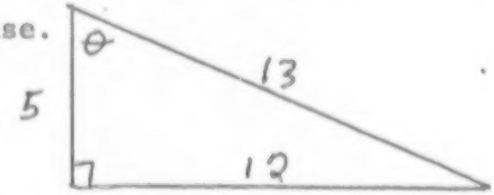
Your two answers should be 41° and 54°

Data Brief # 4



In the above triangle, the side 9" long is opposite the angle θ The side 12" long is adjacent to the angle θ. And the side 15" long is the hypotenuse. The longest side is always the hypotenuse.

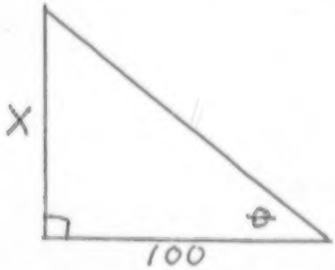
In the triangle below, the side 12' long is opposite angle θ. The side 5' long is adjacent to the angle θ. And the side 13' long is the hypotenuse.



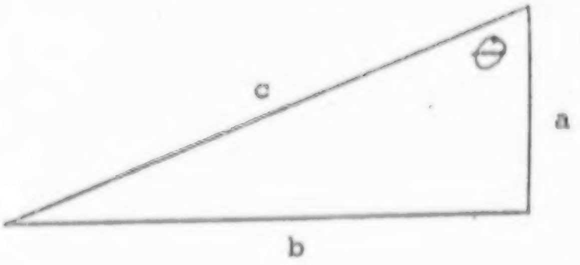
In the triangle below, θ = 40°, the side adjacent to angle θ is 100' long and the side opposite angle θ is x. We want to find x.

$$\tan\theta = \frac{\text{opp.}}{\text{adj.}}$$

$$\tan 40° = \frac{x}{100}$$



To find x you must multiply tan40° times 100. tan 40° = .8391 Thus .8391 times 100 = 83.91 feet. This is the length of the side opposite angle θ,



In the triangle at the left, suppose a = 80 and θ = 60°. Find side b.

$$\tan\theta = \frac{b}{a}$$

$$\tan 60° = \frac{b}{80}$$

$$1.7321 = \frac{b}{80}$$

$$1.7321 \times 80 = b$$

$$138.568 = b$$

Data Brief # 5

For this work assignment you will accompany the teacher out side and will carry out the actual measurement of distances similar to the ones you have completed in this package at this time.

Activity # 1

Tan 40° =

Tan 38° =

Tan 26° =

Tan 67° =

Tan 88° =

Tan 12° =

Tan 17° =

Tan 31° =

Tan 55° =

Tan 0° =

Tan 8° =

Activity # 2

Tan x = .6745, x =

Tan x = .9657 x =

Tan x = .0875 x =

Tan x = .5543 x =

Tan x = 1.0355 x =

Tan x = 1.8040 x =

Tan x = .3640 x =

Tan x = .4452 x =

Tan x = 7.1154 x =

Tan x = .9999 x =

Tan x = .6000 x =

Tan x = .7811 x =

Tan x = .0700 x =

Tan x = 2.144 x =

Activity # 3

Tan x = 6/6 x =

Tan x = 3/4 x =

Tan x = 8/17 x =

Tan x = 4/3 x =

Tan x = 7/3 x =

Tan x = 1/6 x =

Tan x = 2/3 x =

Tan x = 4/9 x =

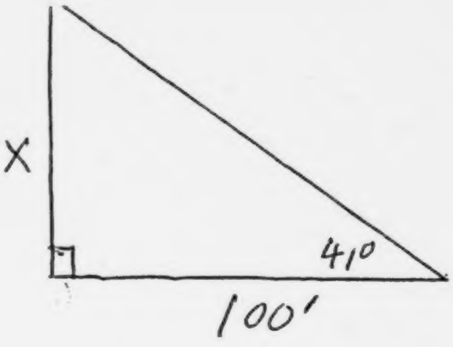
Tan x = 1/17 x =

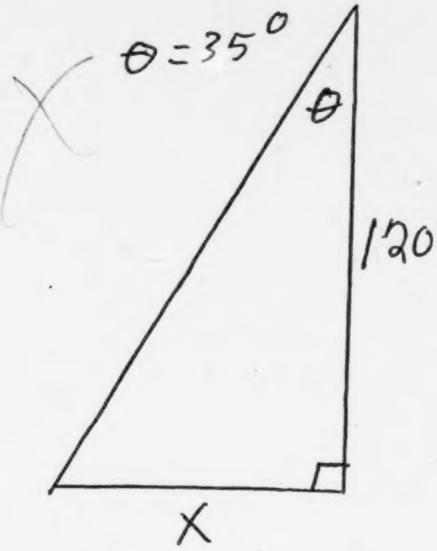
Tan x = 4/5 x =

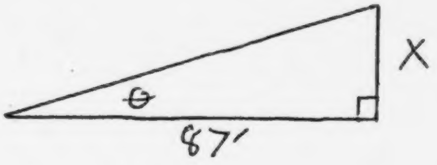
Tan x = 5 x =

Tan x = 2 x =

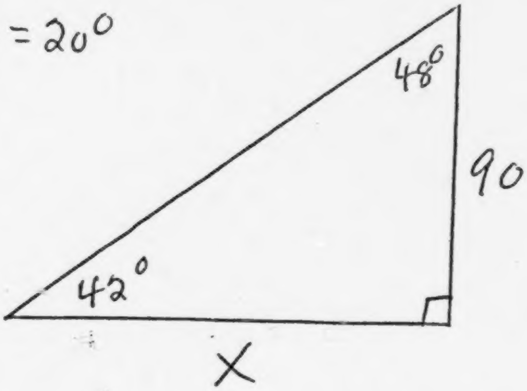
Activity # 4

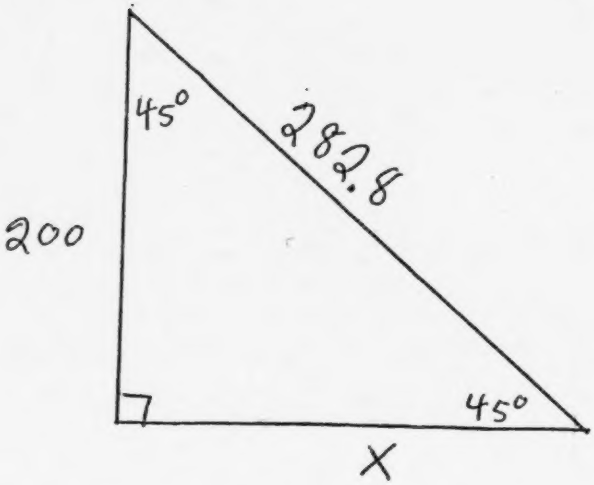


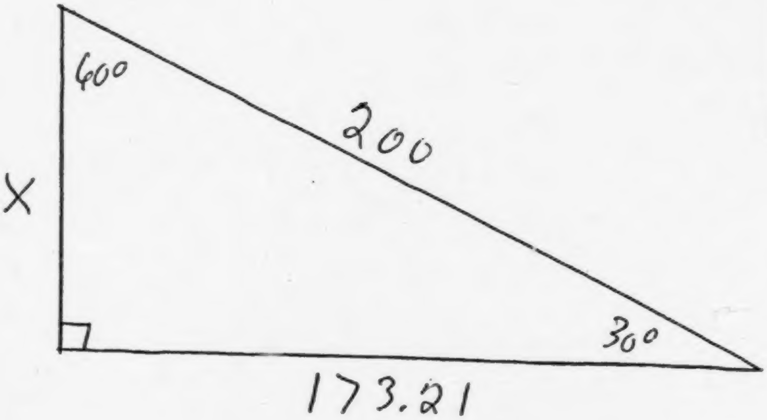




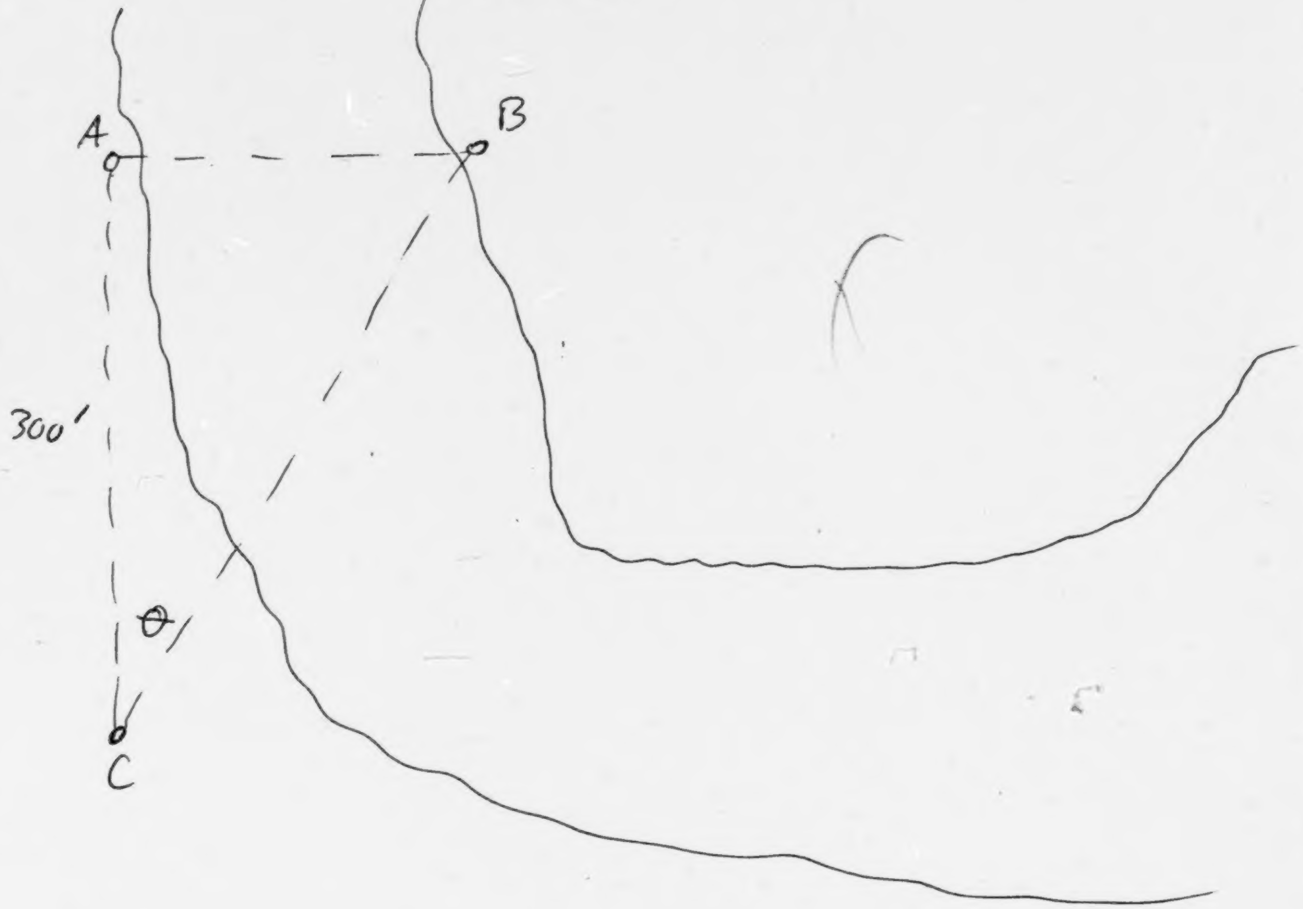
θ = 20°







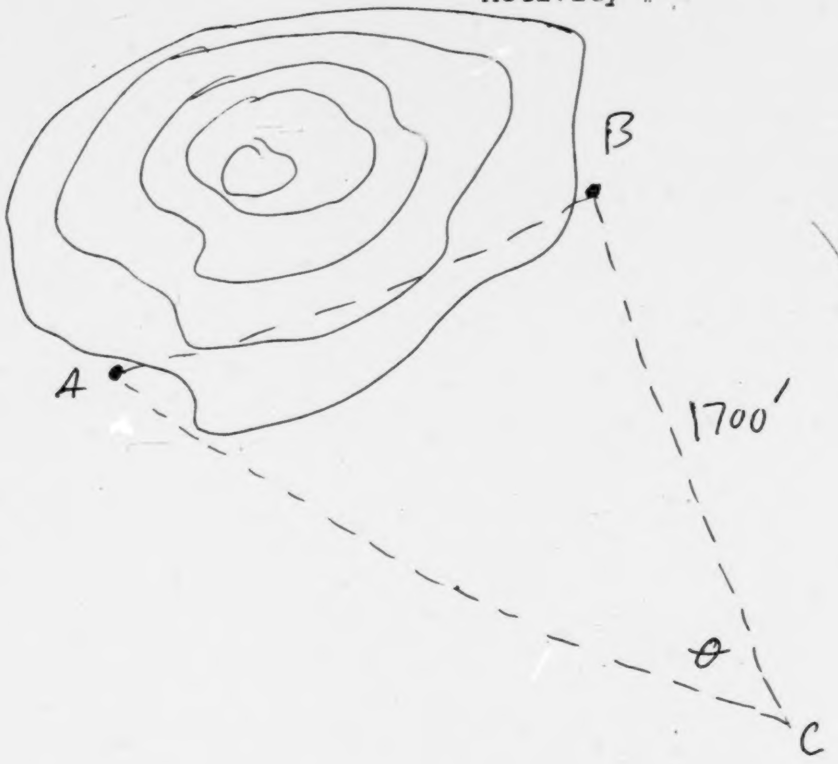
Activity # 4



∇ ανφ ∞ αθε τθεεσ ον οppοσιτε σιφεσ οι α φεερ λοθλ

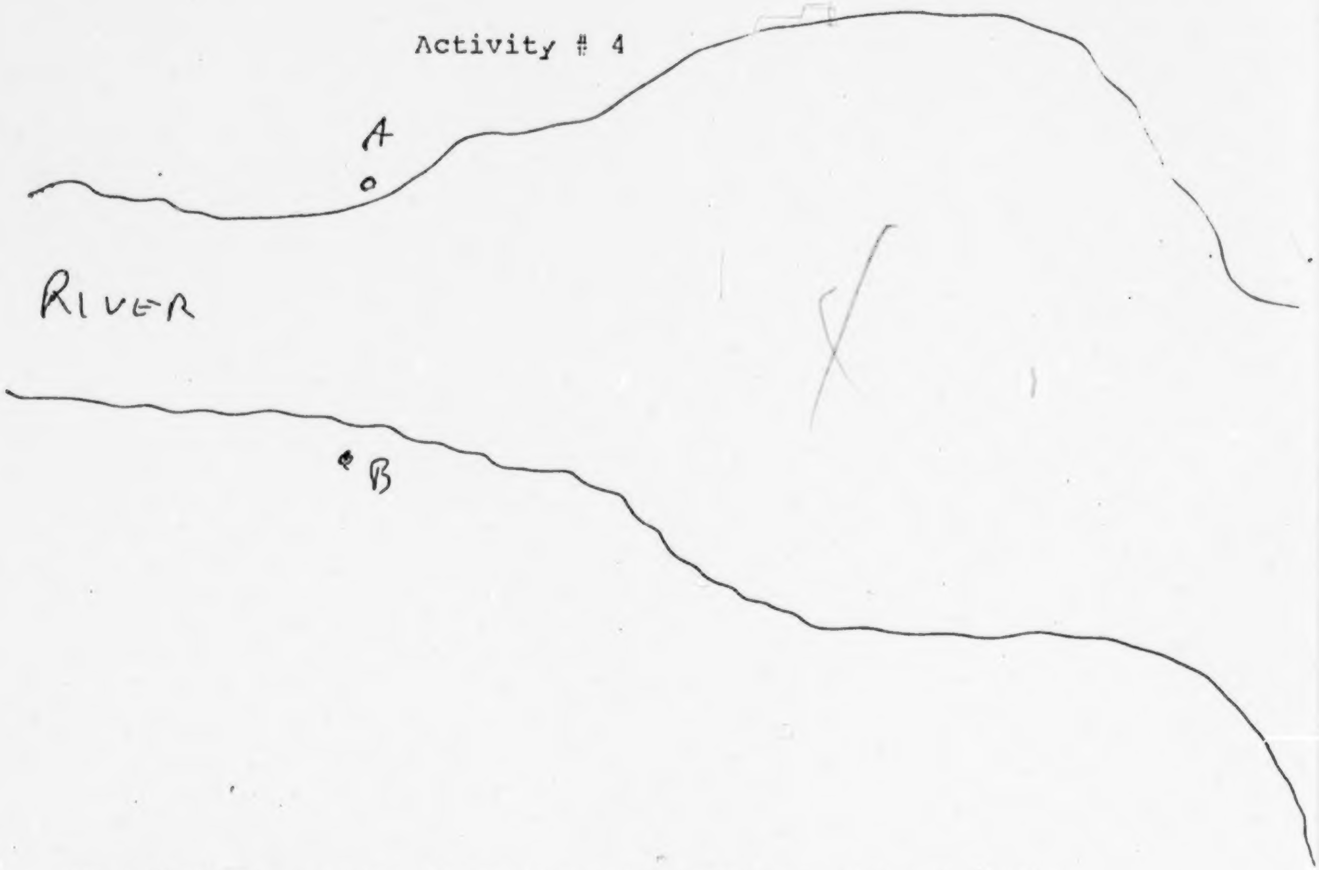
A and B are trees on opposite sides of a deep gorge. Angle A is a right angle. The distance fron A to C is measured and found to be 300'. Angle θ is 38°. What is the distance across the gorge?

Activity # 4



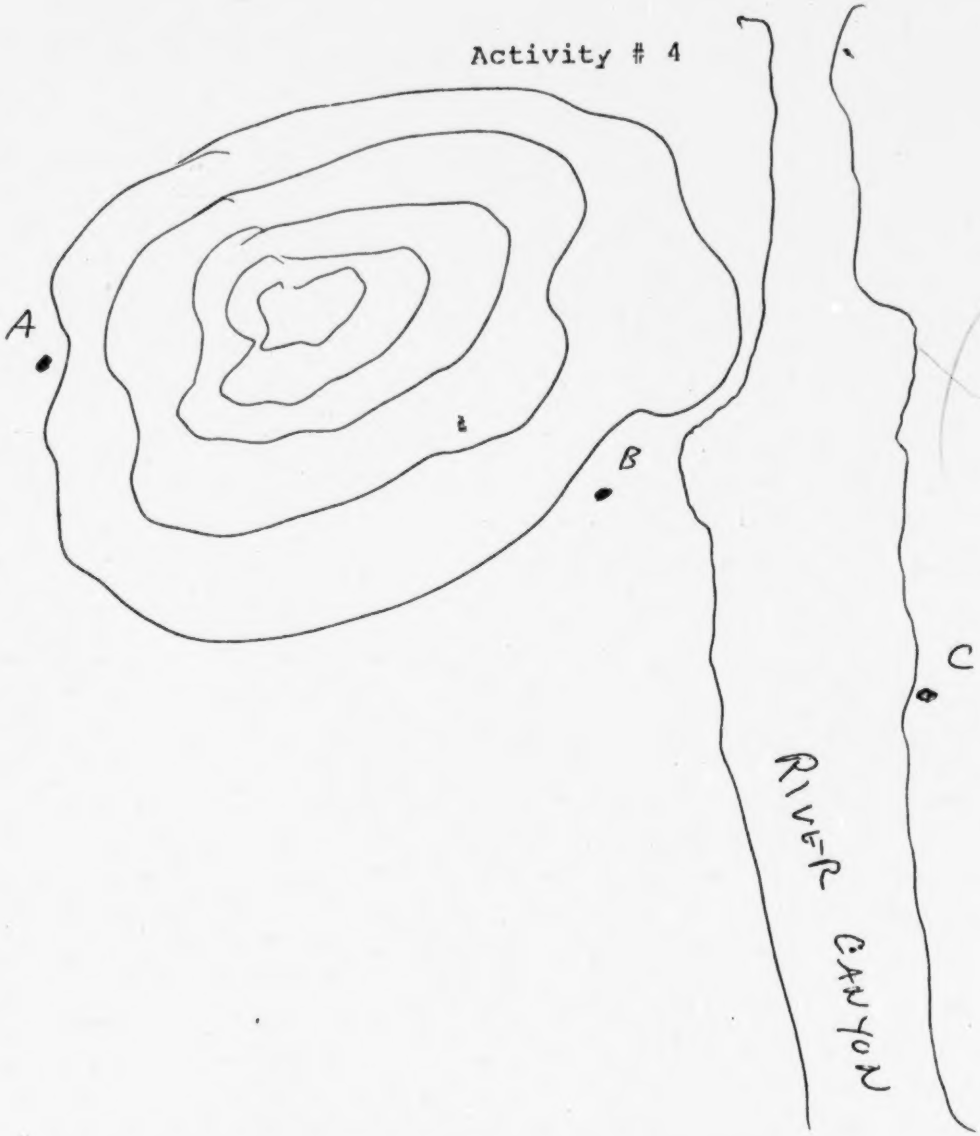
Above is a birds-eye view of a mountain and the surrounding terrain. The railroads want to put a tunnel through from point A to point B. Angle B is a right angle. The distance BC is measured at 1700'. Angle θ is 57°. Find the distance through the mountain.

Activity # 4



The distance across this river in a canyon without wading through the aligator infested waters. You set up the triangle that will allow you to measure this distance and then ask the teacher for the necessary measurements that will let you find the width of the river.

Activity # 4



A tunnel has to go through the mountain from point A to B and then a bridge spans the river canyon from point B to C. You set up the triangles that you will need to measure these distances and then ask the teacher for the necessary measurements that will let you find the distances wanted.

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(STUDENT'S INDIVIDUALIZED CAREER SOURCE PACKAGE)

SERIES EXPLORATION

NUMBER 372.5

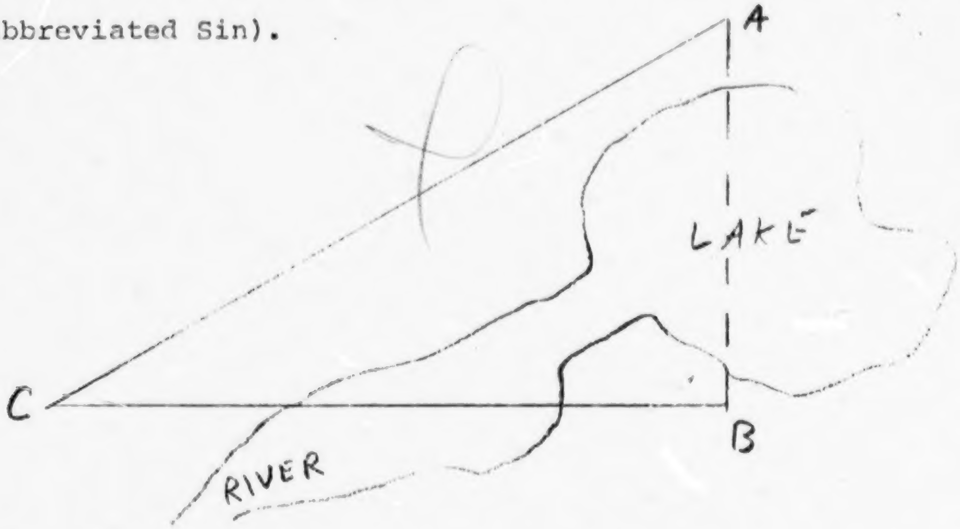
CLUSTER CONSTRUCTION

AREA SITE LAYOUT

TITLE ACROSS A LAKE, ACROSS A RIVER

Rationale:

At times it is not convienient to use the tangent formula because of the difficulty in measuring either of the legs of a right triangle. In the picture below, the distance AB is to be measured, but the distance BC crosses a river and is not easily measured. Therefore the distance AC must be used as a reference distance to find the distance AB. Since the only side of the triangle that is known is the hypotenuse and the side wanted is opposite the angle measured at C, the sine formula will be used instead of the tangent. (abbreviated Sin).



Behavioral Objectives:

Given any angle between 0° and 90°, you will be able to look up its sine exactly.

Given the sine of an angle, you will be able to find the angle with one degree accuracy.

Given a situation similar to the one pictured above, where the side opposite a given angle is to be found and the hypotenuse is known, you will be able to find the distance asked for by using the sine formula, with whole number accuracy.

Pre-Test:

The sin 43° = ?

The sin 67° = ?

The sin x = .5000, x = ?

The sin x = .8660, x = ?

Referring to the picture on the preceeding page of this package -

If the hypotenuse (AC) is 220 yards long, what is the distance AB?

Information Sources:

1. Read Data brief # 1 "The Sine Function and the trig tables."

2. Read Data brief # 2 "The Sine Ratio."

3. Read Data brief # 3 "Using the theodolite."

Data Brief # 1

Look at your trig tables and find out what the sin 30° is. You should get .5000.

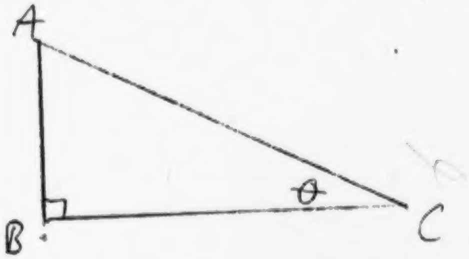
Find the sin 42°. It is .6691

Find the sin 71° It is .9455

If the sin x = .4384, then x = 26°

Data Brief # 2

In the triangle below, side AB is opposite angle . And side AC is the longest side and thus is called the hypotenuse.



The sine formula states that

$$\sin\theta = \frac{OPP}{HYP}$$

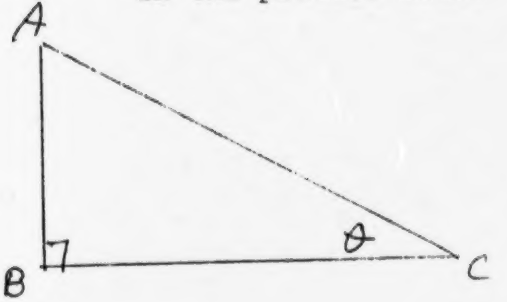
If θ = 23°, and side AC = 80.

Then $\sin 23° = \frac{AB}{80}$

This type of problem is solved by multiplying 80 x (sin23°)

AB = 80 X

In the picture below θ = 37° AB = AC = 230.



$$\sin\theta = \frac{OPP}{HYP}$$

$$\sin 37° = \frac{AB}{230}$$

$$AB = 230(\sin 37°)$$

$$AB = 138.41$$

Data Brief # 3

Get a theodolite and go outside with the teacher to try some actual measurements using the sine formula.

Activity # 1

sin 43° =

sin 56°=

sin 72° =

sin 12° =

sin 30° =

sin 60° =

sin 88° =

sin x = .9945, x =

sin x = .9135, x =

sin x = .4384, x =

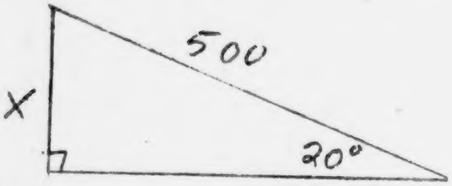
sin x = .1045, x =

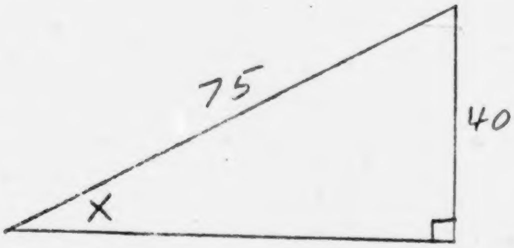
sin x = 4/7 x =

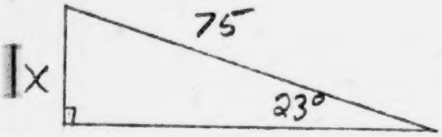
sin x = 5/11 x =

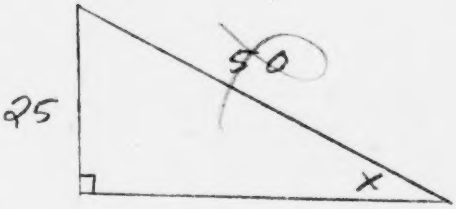
sin x = 9/10 x =

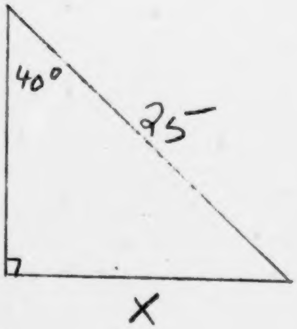
Activity # 2

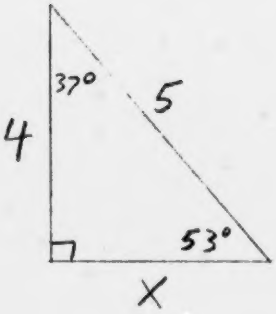

500
X
20°

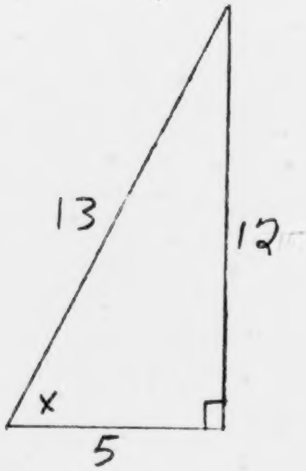

75
40
X

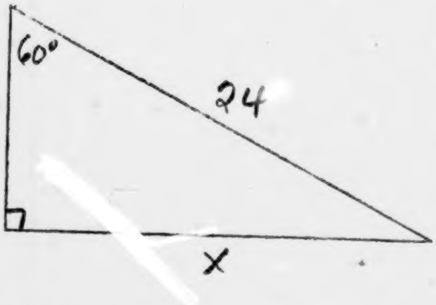

75
X
23°

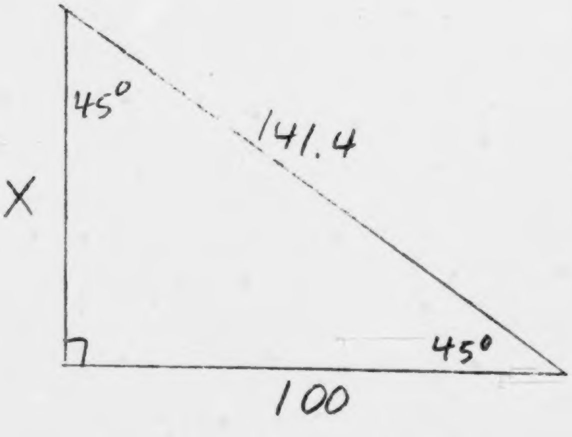

50
25
X

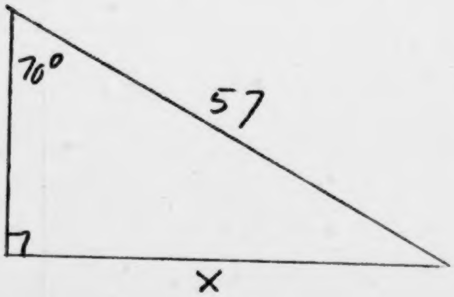

40°
25
X

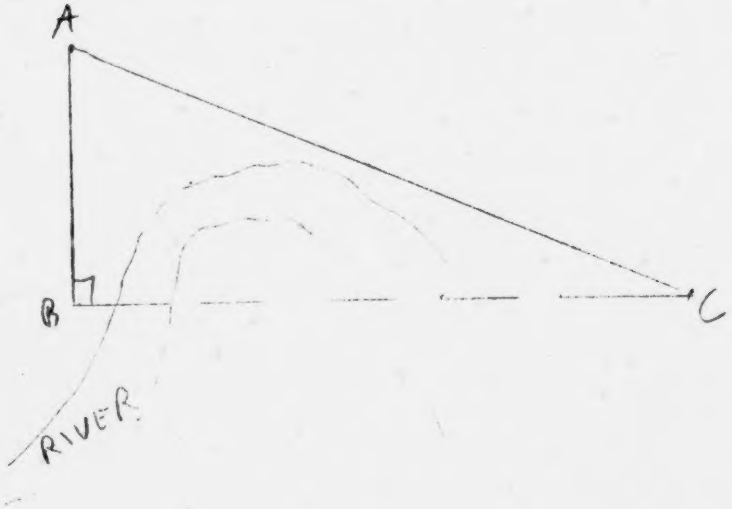

37°
5
4
53°
X


X


60°
24
X


45°
141.4
X
45°
100

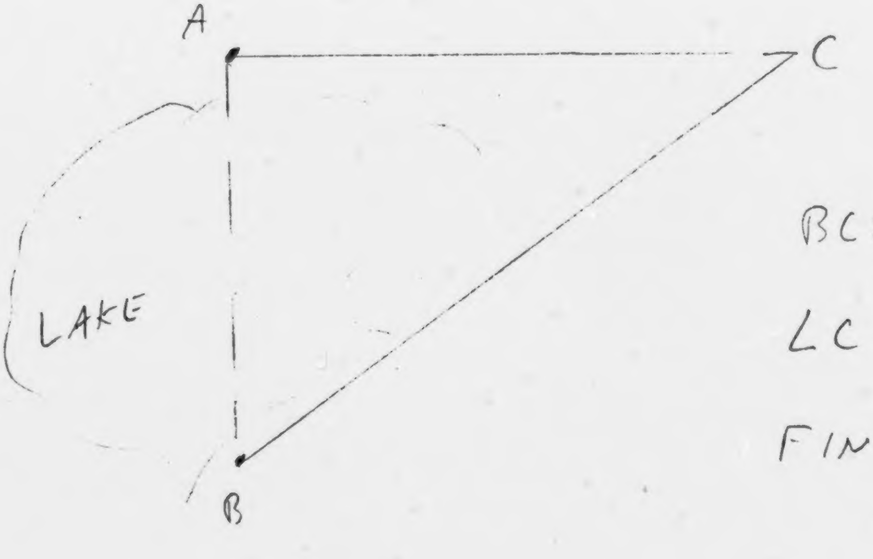

70°
57
X



AC = 225 YDS

$\angle C = 22^\circ$

FIND AB

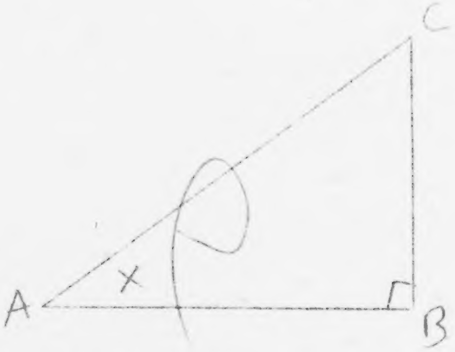


BC = 300 YDS

$\angle C = 41^\circ$

FIND AB

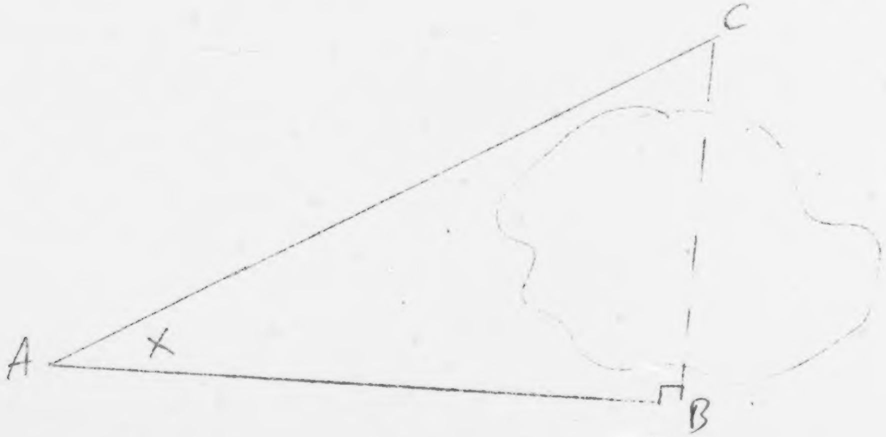
Post Test:



Referring to the triangle at the right. . x = 40°

AC = 120 yds. Find BC

In the above triangle, if x = 26° and AC = 400 ft. Find BC



The length of a property line has to be measured from B to C, across a small pond.

If AC = 200 ft. and x = 30°

Find BC.

S. I. C. S. PAK

(STUDENT'S INDIVIDUALIZED CAREER SOURCE PACKAGE)

SERIES: Exploration

A

NUMBER: 372.6
CLUSTER: Transportation
AREA: General Aviation
TITLE: Who's Going Fasteast

Rationale:

Just because a plane is flying at 400 mph doesn't mean he will travel a distance of 400 miles from one place on the earth to another in one hour. Maybe he's flying 400 mph straight up. This means his ground speed is different than his air speed.

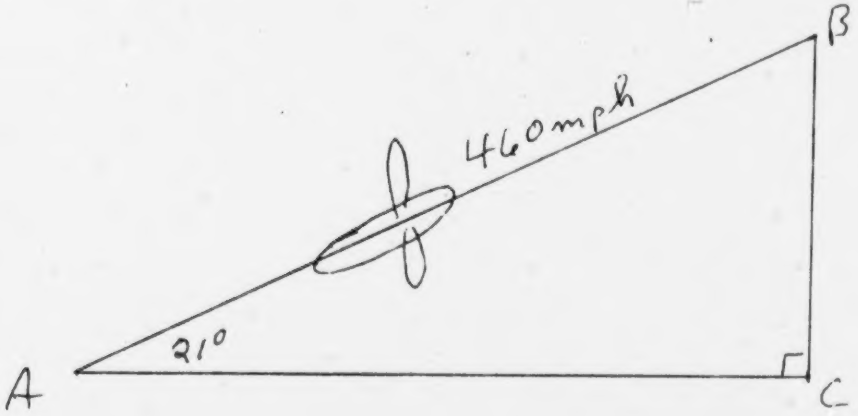
This package will demonstrate a method for computing ground speed if the angle of elevation and air speed are known.

Behavioral Objectives:

Given the air speed and angle of elevation of an airplane you will be able to calculate its ground speed with whole number accuracy.

Pre-Test:

Find the ground speed of the airplane in the picture below.



Pre-Test (continued)

An airplane is climbing at an angle of 16° at a speed of 375 mph. What is its ground speed?

Information Sources:

1. Read Data Brief # 1 "Cosine and the trig tables"
2. Read Data Brief # 2 "Triangles and Cosine"
3. Read Data Brief # 3 "Using the theodolite"

Data Brief # 1

Use your trig tables for looking up the cosine of an angle the same way you did for looking up sine and tangent.

Example: Cos 10° = .9848

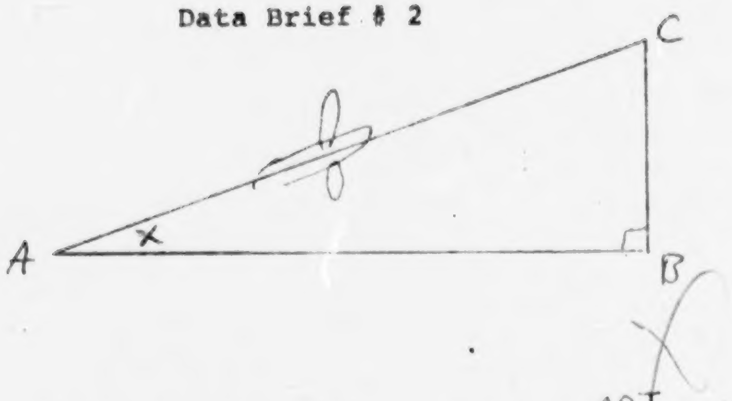
Cos 60° = .5000

Cos 30° = .8660

Cos x = .8192 x = 35°

Cos x = .7075 x = 45°

Data Brief # 2



The cosine formula says $\cos x = \frac{ADJ}{HYP}$

Referring to the picture above, Line AC describes the path of an airplane climbing at an angle x. Line AC represents the speed of the airplane, not the distance it is flying. Thus, line AB represents its ground speed.

Gicen that the airplane is climbing at an angle of 25° at a speed of 200 mph, Its ground speed is found in the following manner.

$$\cos x = \frac{ADJ}{HYP}$$

$$\cos 25° = \frac{AB}{200}$$

$$.9063 = \frac{AB}{200}$$

$$181 = AB$$

The ground speed is 181 mph.

Data Brief # 3

Pick up a theodolite and a teacher and go outside for some actual measuring with Cosine.

Activity 0 1

Cos 32° =

Cos 47° =

Cos 76° =

Cos 45° =

Cos 10° =

Cos 89° =

Cos 67° =

Cos x = .1564 x =

Cos x = .7193 x =

Cos x = .8910 x =

Cos x = 6/7 x =

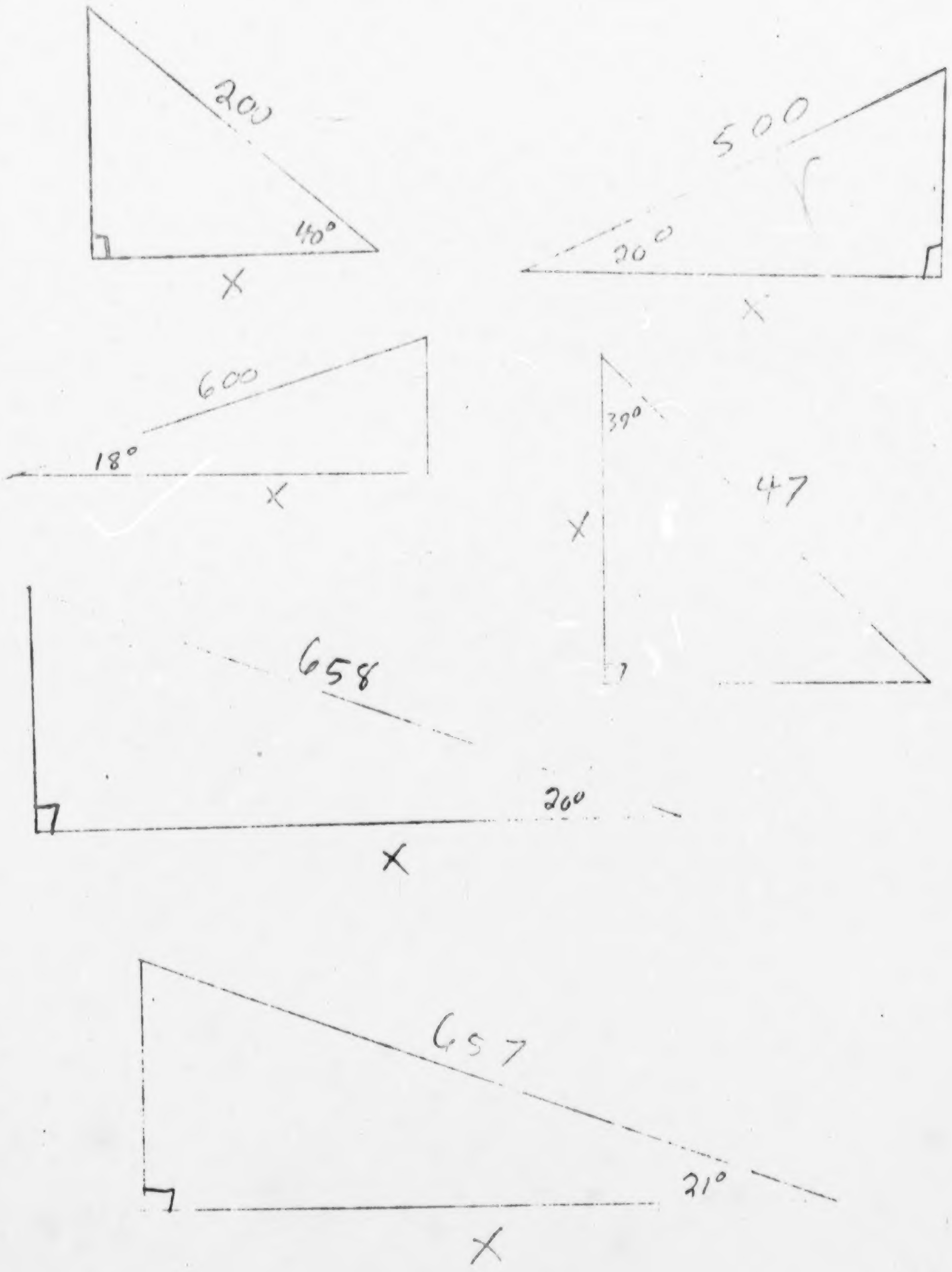
Cos x = 1/4 x =

Cos x = 7/9 x =

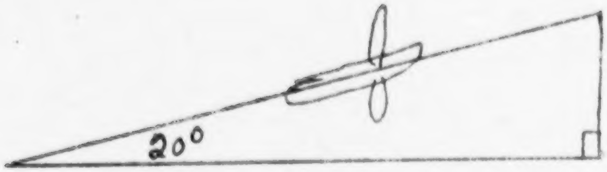
Cos x = 347/456 x =

Cos x = 23/450 x =

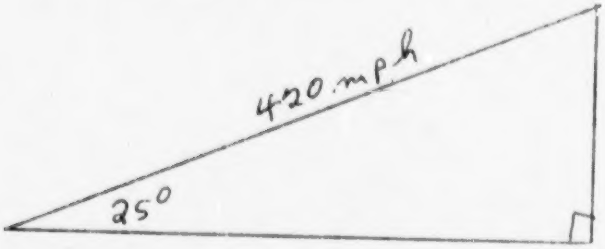
Activity # 2



Activity # 2



Find the ground speed of the airplane in the above picture if the air speed is 350 mph, and the plane is climbing at an angle of 20°



Find the ground speed of the airplane from the above picture.

An airplane is climbing at an angle of 29° at a speed (air speed) of 400 mph. What is its ground speed?

An airplane is climbing at an angle of 15° with an air speed of 275 mph. What is its ground speed?

S. I. C. S. PAK

(STUDENT'S INDIVIDUALIZED CAREER SOURCE PACKAGE)

SERIES: Exploration

NUMBER: 372.7
CLUSTER: Travsportation
AREA: Data Handling
TITLE: Where Are You

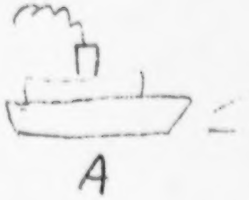
Rationale:

At times when you are on the ocean or in the air, it is helpful to know how far away different objects are. It is up to the navigator to decide which trig formula to use: sine, cosine, or tangent. In this package you are the navigator and you will pick which formula to use to get the information asked for.

Behavioral Objectives:

Given different situations involving indirect measurement by use of the sine, cosine, and tangent formulas, you will be able to pick which of the three formulas to use and find the distance with whole number accuracy.

Pre-Test:



Referring to the above picture:

When boat A is due west of the lighthouse and boat B is due north, the man in the light house tells the navigator of boat A that he is 5000 ft. from the lighthouse. At this instant the navigator measures the angle from the lighthouse to boat B and finds it is 32°.

What formula would you use to find the distance from the lighthouse to boat B? ____________________

What formula would you use to find the distance from boat A to the boat B? ____________________

Find both these distances.

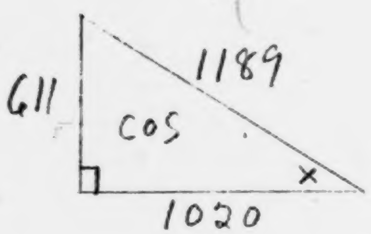
Information Sources:

1. Read Data brief # 1 "Pick your formula - angles"
2. Read Data brief # 2 "Pick your formula - sides"
3. Read Data brief # 3 "Using the theodolite"

Data Brief # 1

In each problem on activity sheet # 1, you are to find the measure of the angle designated by x, using the formula noted inside the triangle.

example:



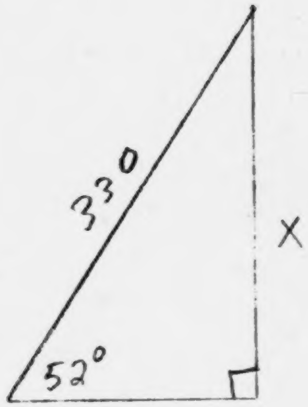
$$\text{Cos } x = \frac{ADJ}{HYP}$$

$$\text{Cos } x = \frac{1020}{1189}$$

$$\text{Cos } x = .8579$$

Using the trig tables, x = 31° (approximately)

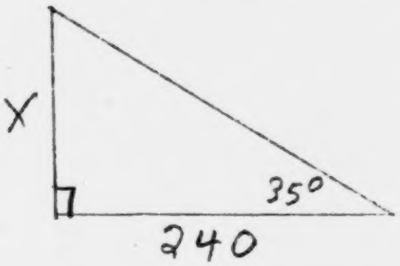
Data Brief # 2



In the above triangle, the side opposite the known angle is the side wanted and the hypotenuse is known. For this set of data the Sine formula would be the easiest to use because

$$\text{Sin } x = \frac{OPP}{HYP}$$

$$\text{SIN } 52^\circ = \frac{X}{330}$$



In the above triangle, you are given the side adjacent to the known angle and you want the side opposite the known angle. Therefore the easiest way to solve for x would be to use the tangent formula because

$$\tan \theta = \frac{\text{OPP}}{\text{ADJ}}$$

$$\tan 35° = \frac{X}{240}$$

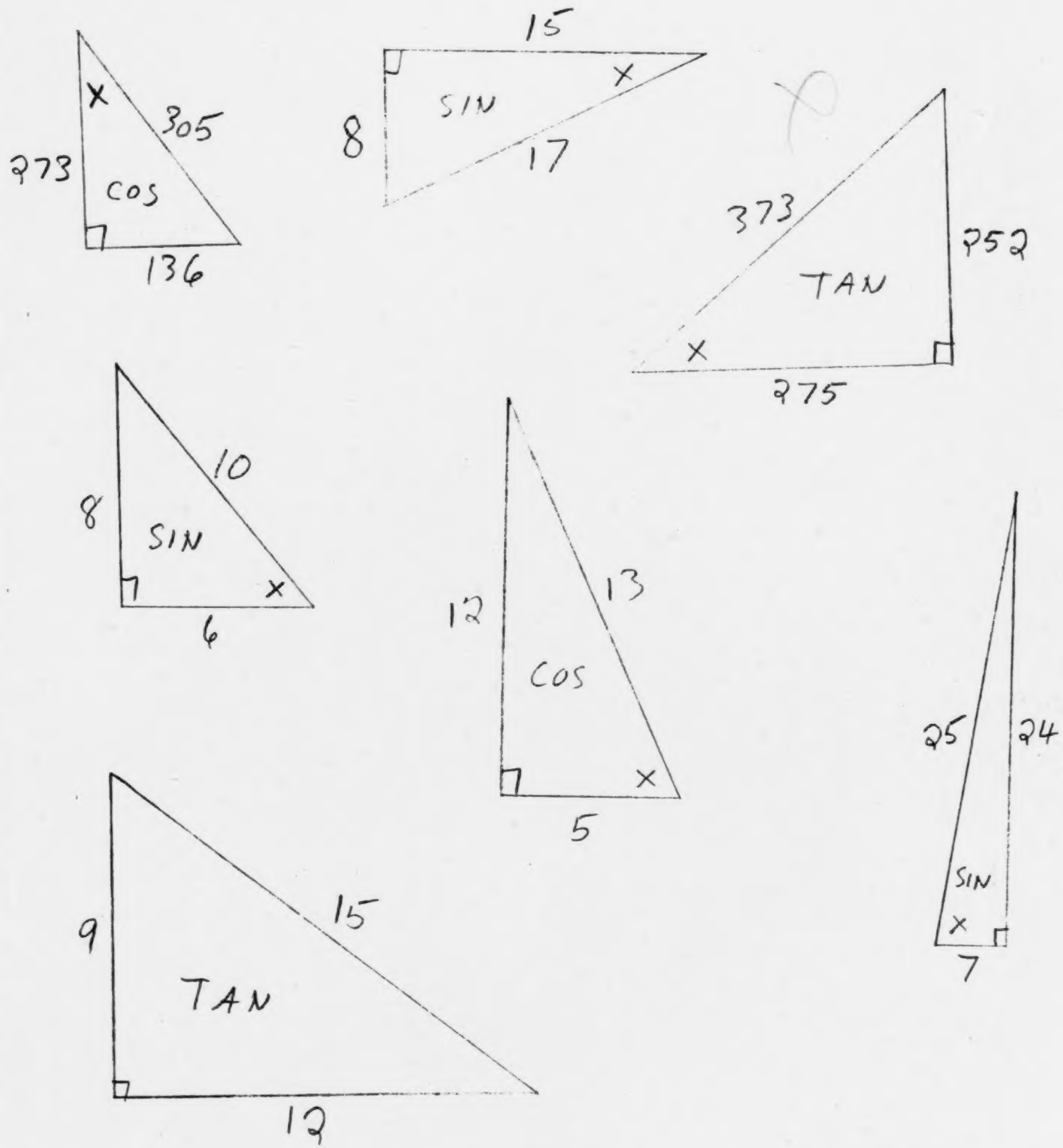
$$.7002 = \frac{X}{240}$$

Data Brief # 3

Pick up a tape measure, a theodolite and a teacher and go out side for some practical problems.

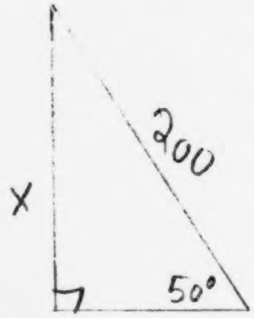
Activity # 1

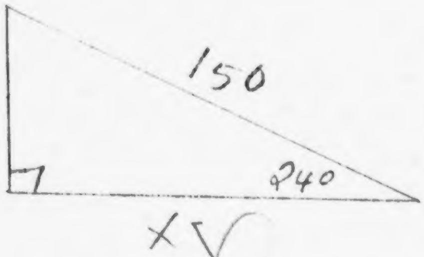
Use the trig formula mentioned to find the angle x.

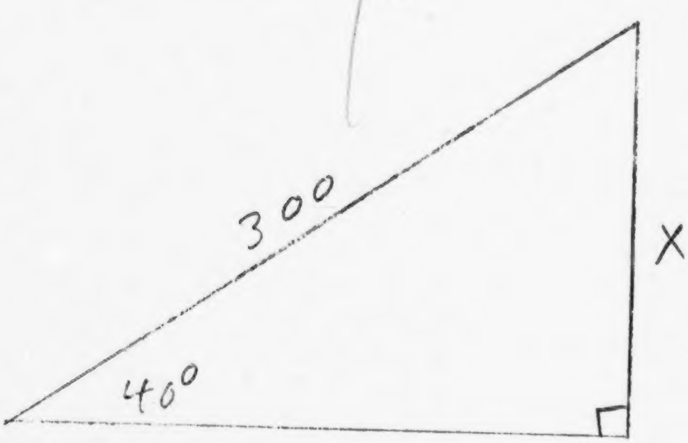


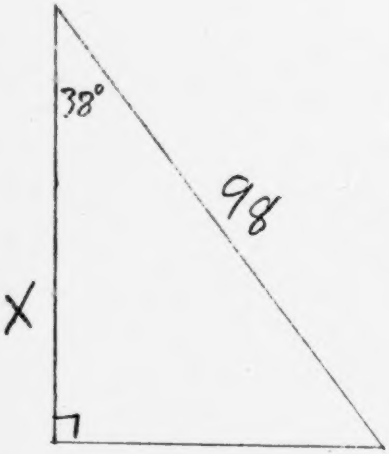
Activity # 2

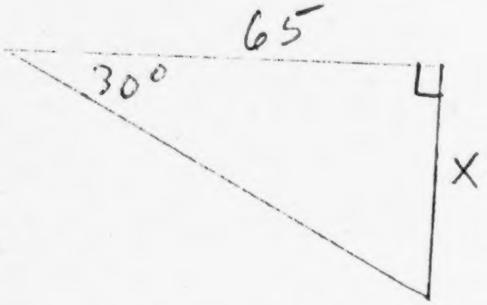
Name the trig formula you are going to use and show it to the teacher before solving for the unknown.

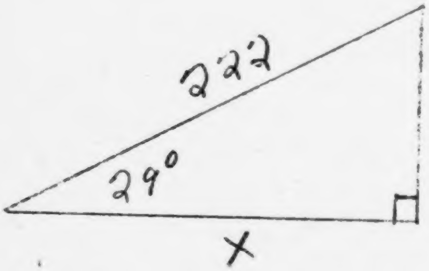


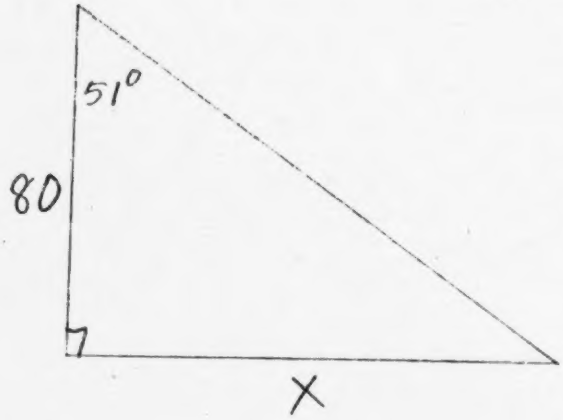


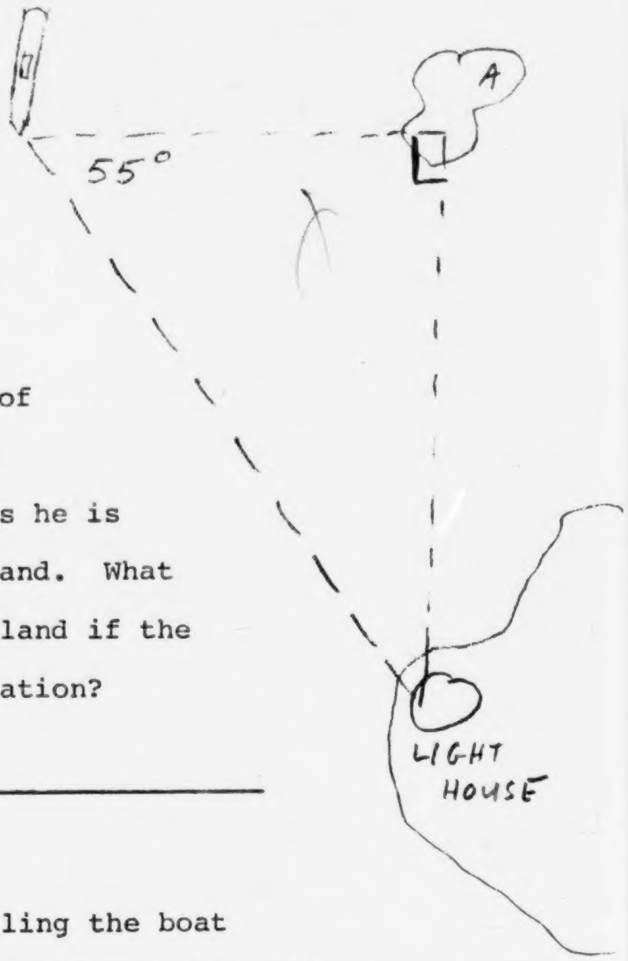












Because of the shallow water about island A, the ship must stay in a channel 1200' away from the island. The man in the lighthouse radios to the captain of the ship and tells him that he is 2092' from the lighthouse, just as he is passing directly opposite the island. What is the ships distance from the island if the picture above represents the situation? Which trig formula????

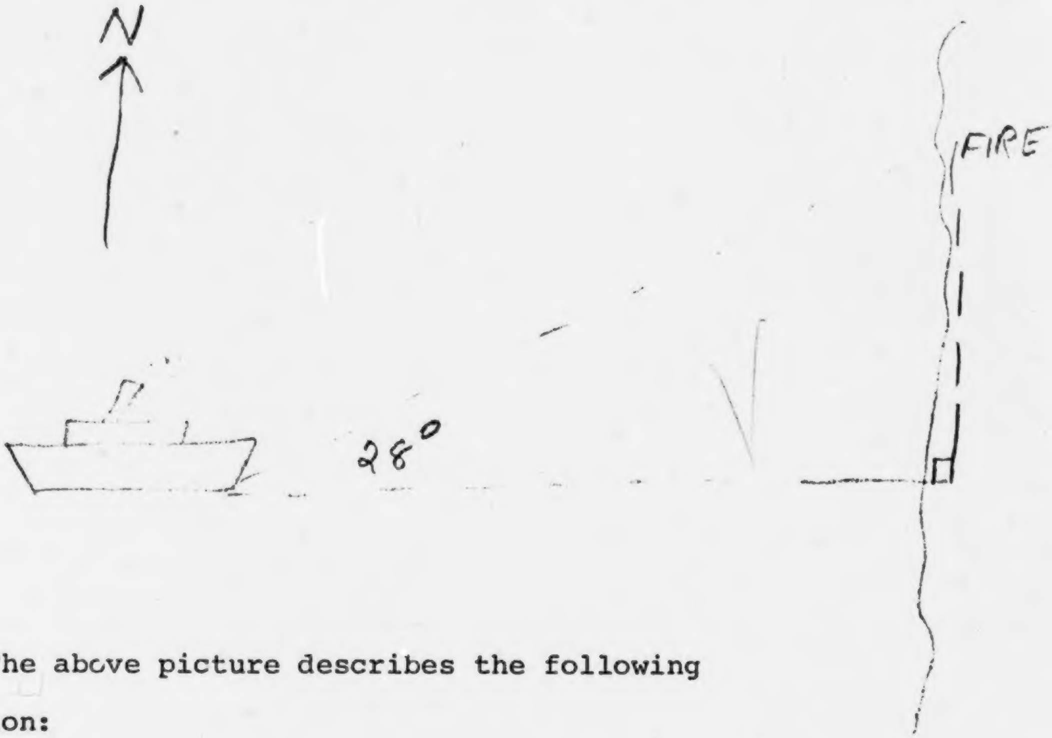
____

Suppose that instead of telling the boat captain how far he was from the lighthouse, he told him how far the island was from the light house. Which trig formula would the ship's navigator use then?

____

How far is the island from the lighthouse?

____



The above picture describes the following situation:

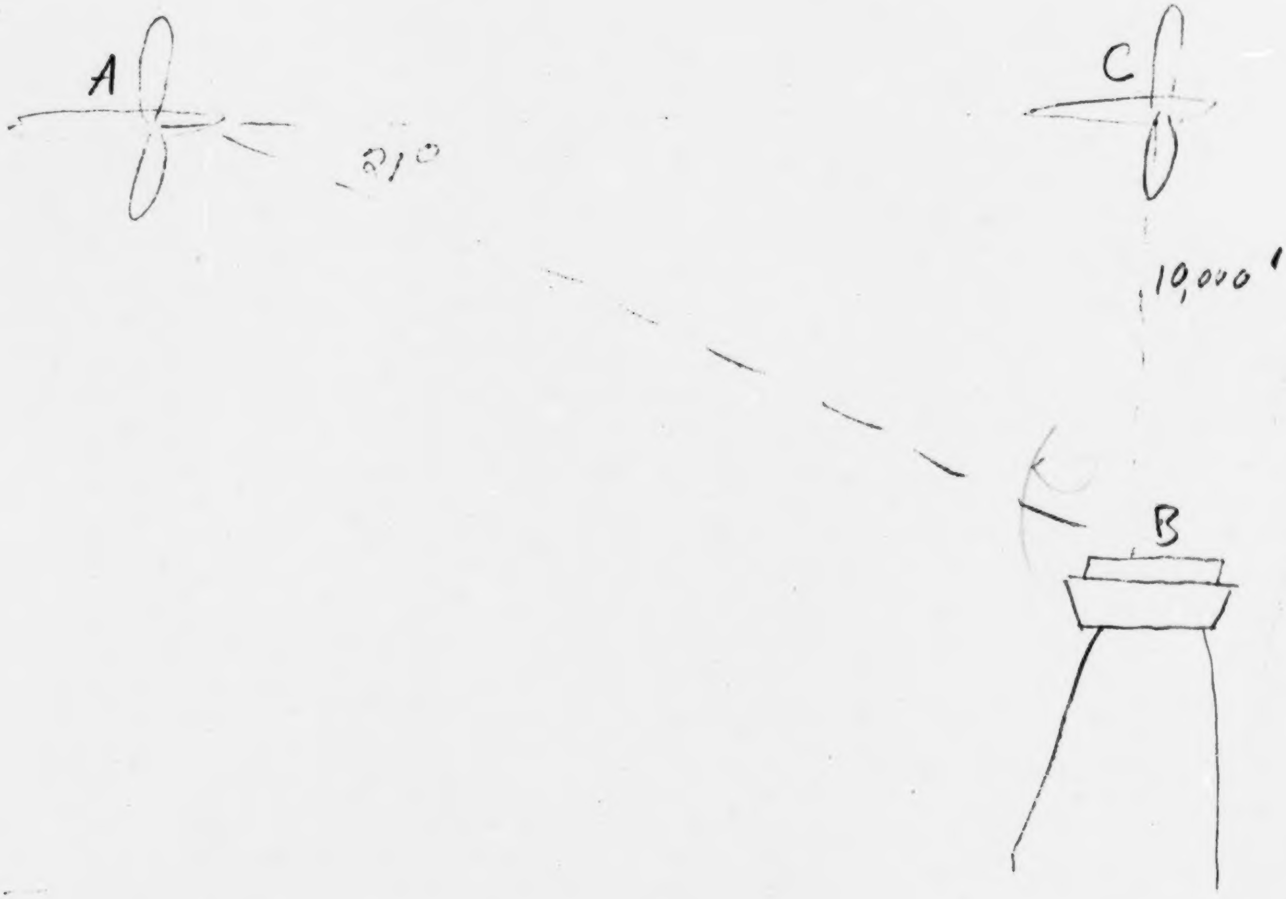
A ship heading due west out of port sees a cloud of smoke on the shore line due north of the port, at an angle of 28°. He knows he is 16,000' from the shore (about 3 miles). How far up the coast is the fire?

____

Which trig formula??????

Which trig formula would he use to find out how far the ship is from the fire?

How far is the ship from the fire? __________________



Two airplanes are flying at an altitude of 10,000 feet. The navigator in plane A is looking through his theodolite and measures an angle of 21° just as the tower tells plane C that he is directly overhead.

What trig formula would you use to find the distance between the airplanes?

How far apart are the airplanes? ________________

What trig formula would you use to find how far plane A is from the tower (line AB).

How far is plane A from the tower? ________________

S. I. C. S. PAK

(STUDENT'S INDIVIDUALIZED CAREER SOURCE PACKAGE)

SERIES: EXPLORATION

NUMBER: 374.8

CLUSTER: PUBLIC SERVICE

AREA: PUBLIC RECORDS, TITLES AND DEEDS

TITLE: THE $25,000 CAPER

Rationale:

Even a real estate broker or an escrow officer needs to know math, and not just simple arithmetic. This package leads up to the solving of a problem that actually came up in a Tucson real estate office. The solution to the problem was worth $25,000 to the owner of the real estate office.

But even after the solution was presented to him, the real estate man failed to capitalize on the information because he didn't understand the math involved which was the use of the Law of Cosines and the solving of a second degree equation.

Behavioral Objectives.

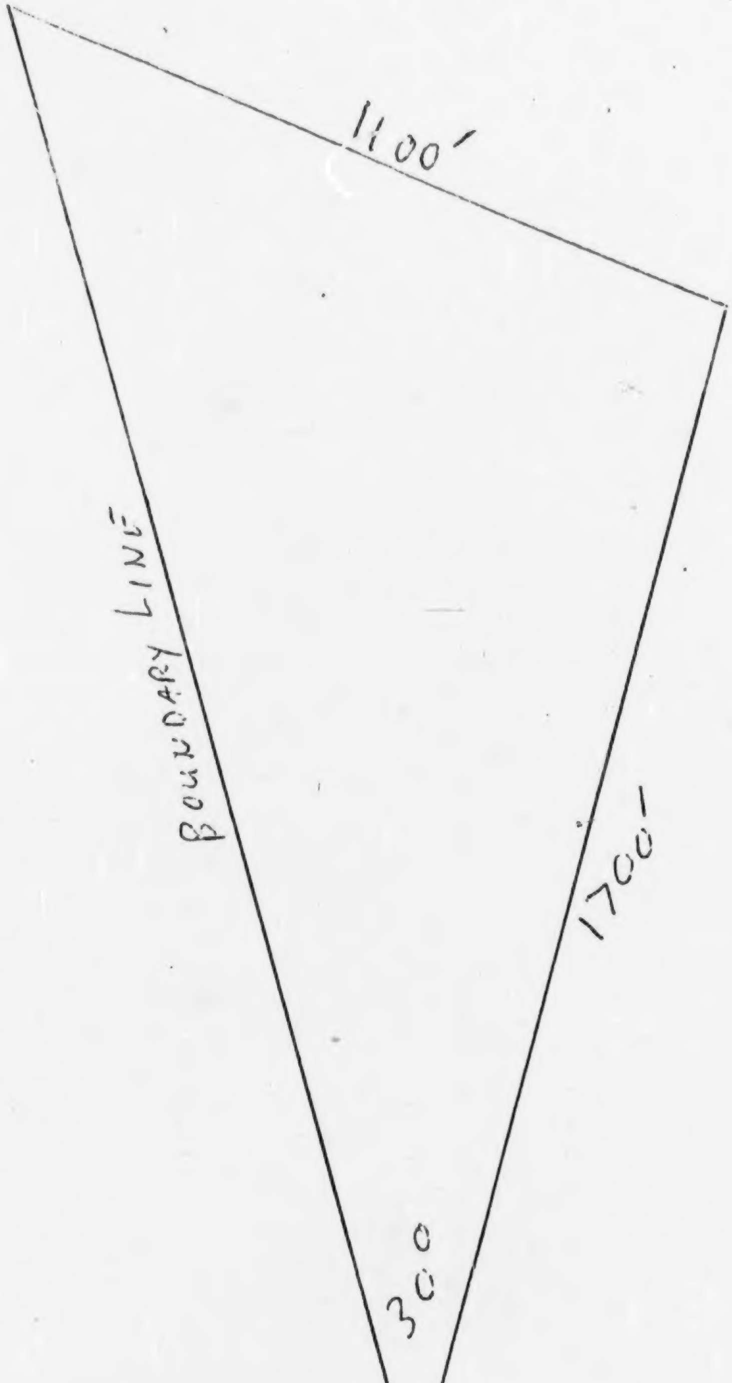
Given a problem to solve that requires the use of the Law of Cosines, you will be able to set up the equation and solve the problem with 3 decimal place accuracy, with the use of the calculator and trig tables.

Information Sources:

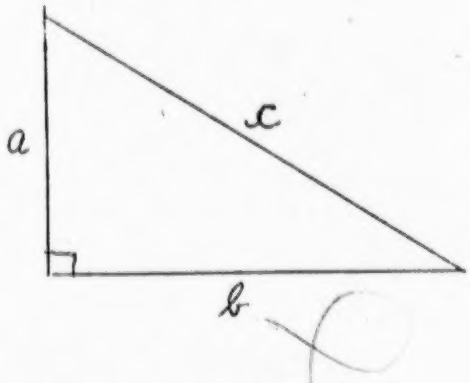
Read Data brief #1	"The Law of Cosines"
Read Data brief #2	"The Quadratic Formula"
Read Data brief #3	"For $25,000, what is x"
Read Data brief #4	"The theodolite and the Law of Cosines"

PRE TEST

FIND THE LENGTH OF THE BOUNDARY LINE IN THE PICTURE BELOW, USING THE LAW OF COSINES. THE DRAWING IS ACCURATE.



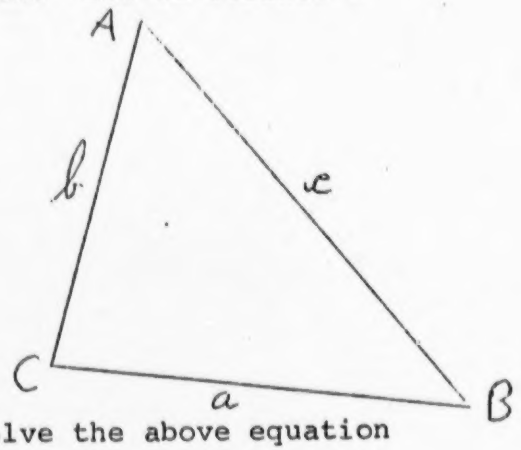
Data Brief #1



The Pythagorean theorem states that in any right triangle, as in the picture to the right

$$a^2 + b^2 = c^2$$

There is also a "Pythagorean theorem" for any triangle, not just right triangles. It is called the "Law of Cosines".



Using the triangle at the right as an example, this law states that:

$$c^2 = a^2 + b^2 - 2abCosC$$

Suppose that angle C = 90°. Then solve the above equation for c if a = 3, b = 4. (Remember that Cos90° = 0.)

You should get c = 5. This illustrates that the pythagorean theorem can be developed from the Law of Cosines.

If you have to find side b in a triangle the Law of Cosines states that

$$b^2 = a^2 + c^2 - 2acCosB$$

If you want to find side a the law states that

$$a^2 = c^2 + b^2 - 2bcCosA.$$

Data Brief # 2

The quadratic formula allows you to solve any second degree equation by plugging numbers into the formula and grinding out the answer.

The formula states that if $ax^2 + bx + c = 0$ then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a is the coefficient of the x^2 term, b is the coefficient of the x term and c is the constant.

Given the equation $4x^2 + 6x - 7 = 0$ to solve,

$a = 4$

$b = 6$

$c = -7$

Given the equation $x^2 + 3x = 13$ to solve,

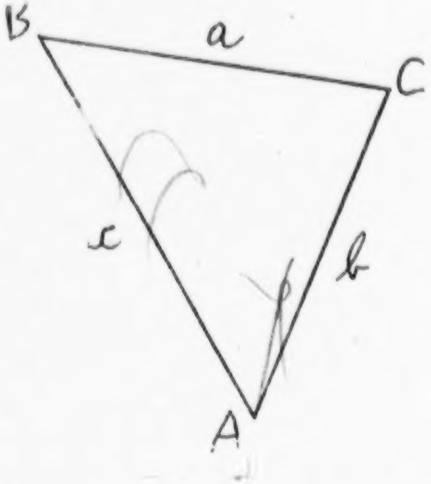
$a = 1$

$b = 3$

$c = -13$

This is not a mistake, $c = -13$. To put this equation in the proper form to be solved by the quadratic formula the +13 has to be brought over to the other side of the equal sign.

At this point you shoud understand that the a,b,c in the quadratic formula have nothing to do with the a,b,c in the Law of Cosines.



In the triangle at the right, if the drawing is accurate find side c if $a = 4$, $b = 3$ and $A = 62°$.

Since you are given angle A you have to use the Law of Cosines with an a by itself on one side of the equal sign as illustrated below.

$$a^2 = b^2 + c^2 - 2bc\text{Cos}A$$

$$16 = 9 \quad c^2 - 2(3)(c)\text{Cos}62°$$

$$0 = -7 + c^2 \quad -6(.4695)$$

$$0 = -7 + c^2 - 2.817c$$

Since we want to solve this equation for c, we will change the letter c to an x. Thus,

$$0 = -7 + x^2 - 2.817x$$

$$0 = x^2 - 2.817x - 7$$

Thus $a = 1$

$b = -2.817$

$c = -7$

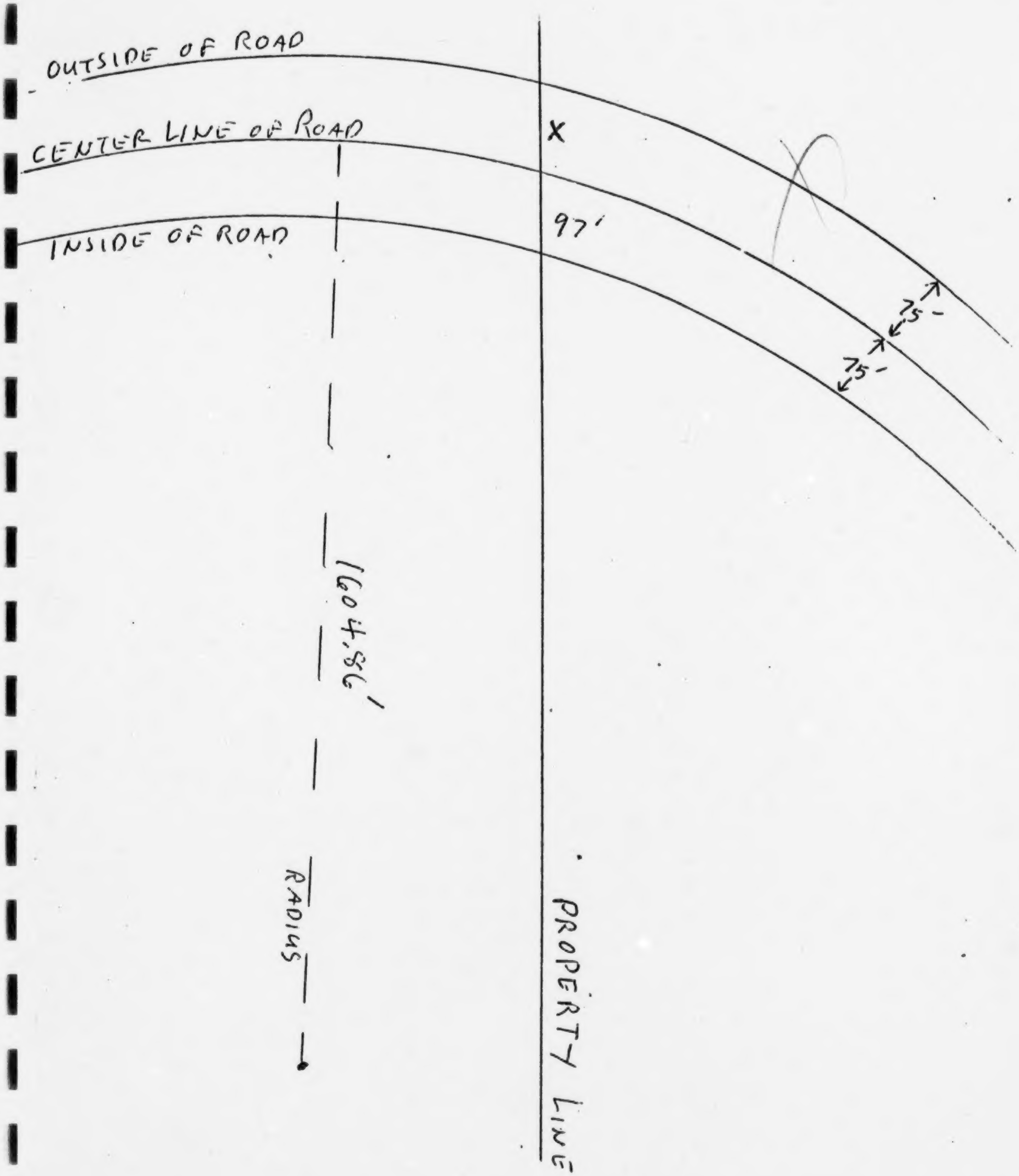
Data Brief # 3

The following page of this package has a drawing of the situation. As you read this page look at the picture and you will understand the problem better.

A curved road cuts across a piece of property. The curved road forms part of the circumference of a circle, and the radius of that circle is known: 1604.86 ft. This radius is measured from the center of the road to the center of the circle. The road is 150 ft. wide altogether. The edges of the road and the center line form three concentric circles. The property line is cut by the three concentric circles just as a transversal would be cut by three parallel lines an equal distance apart. The problem is: do the three concentric circles cut the property line into two equal parts? The property line does not go through the (imaginary) center of the circles, but is fairly close to it. In the drawing accompamying this problem, the drawing of the curve of the road and the width of the road are not drawn to scale. What you have to find is Does $x = 97$ ft.

Your first job is to prove toa non-mathematician who had high school geometry 40 years ago, that either the boundary line is cut into equal parts or it isn't, by the circles. Then prove your answer by finding the length of x.

The company that originally drew up the deed for this property assumed that x was 97 ft. The real estate man felt that if it was shorter then his property along the side of the road had been encroached on and he could probably get an extra $25,000 from the county highway department because of this.


OUTSIDE OF ROAD
CENTER LINE OF ROAD
INSIDE OF ROAD
X
97'
75'
75'
1604.56'
RADIUS
PROPERTY LINE

Data Brief # 4

With a theodolite and a measuring tape, you will accompany the teacher outside to actually use the law of Cosines.

Activity # 1

Using the Law of cosines find the information requested below.

Find a if $b = 6$ $c = 4$ $A = 60°$

Find c if $b = 3$ $a = 5$ $C = 40°$

Find b if $a = 10$ $c = 1$ $B = 30°$

Find c if $b = 2$ $a = 6$ $c = 20°$

Find a if $b = 2$ $c = \sqrt{3}$ $A = 39°$

Find c if $b = \sqrt{2}$ $a = 7$ $C = 45°$

On the

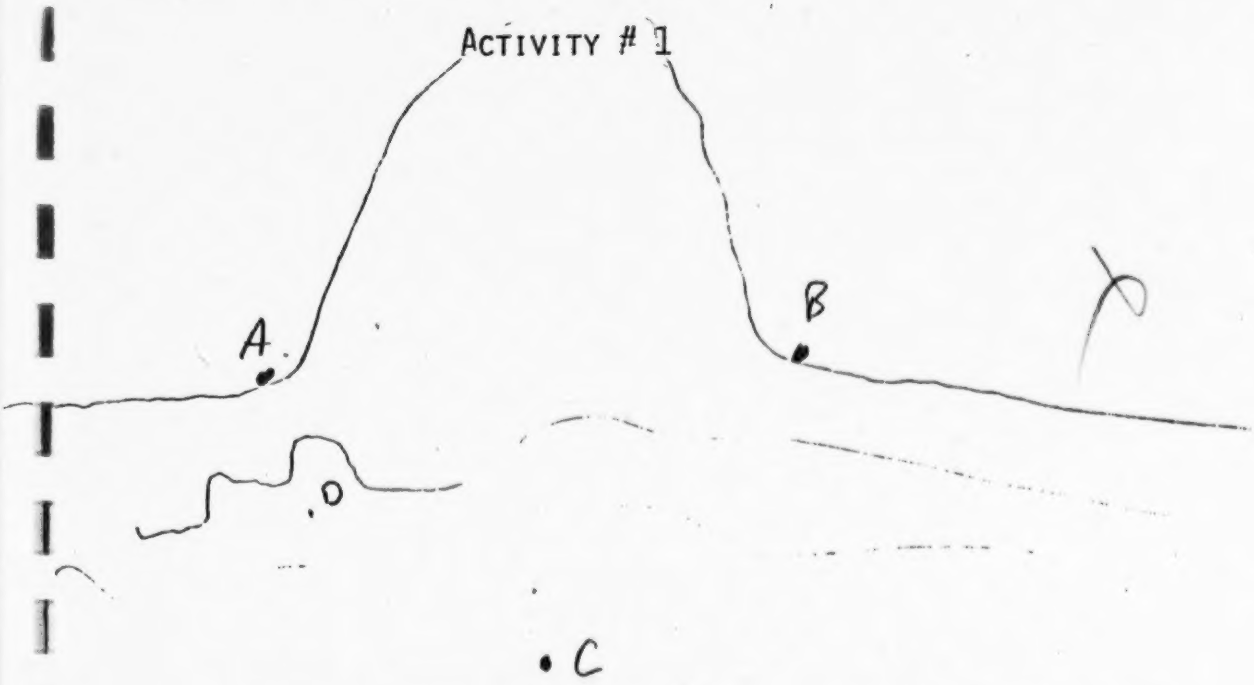
For the two problems below, set them both up and show them to the teacher BEFORE doing any arithmetic.

Find c if $a = 2$ $b = 4$ $A = 60°$

Find c if $a = \sqrt{2}$ $b = \sqrt{3}$ $A = 60°$.

1. A surveyor at C sights two points A and B on opposite sides of a lake. If C is 5000 ft. from A and 7500 ft. from B, and angle ACB measures 30°, how wide is the lake?

ACTIVITY # 1



A RAILROAD COMPANY WANTS TO TUNNEL THRU A MOUNTAIN FROM POINT A TO POINT B. TO MAKE AN ACCURATE ESTIMATE AS TO COST, THEY HAVE TO KNOW HOW FAR IT IS FROM A TO B. THE ONLY PRACTICAL PLACE TO SET UP A THEODOLITE IS AT POINT C.

AC MEASURES 1350 YARDS
BC MEASURES 1926 YARDS
ANGLE C = 72°
WHAT IS THE DISTANCE FROM A TO B. ?

EXPLAIN HOW YOU WOULD DO THIS PROBLEM IF THE HILL AT POINT D BLOCKED YOUR SIGHT OF POINT A, AND YOU CAN FIND NO OTHER PLACE TO PUT THE THEODOLITE.

Activity # 2

Solve the following equations for all possible values of x using the quadratic formula where necessary.

1. $x^2 + 7x + 12 = 0$
2. $x^2 + 9x + 14 = 0$
3. $x^2 + 3x - 70 = 0$
4. $x^2 - 6x + 8 = 0$
5. $x^2 - 10x + 9 = 0$
6. $x^2 - 11x - 26 = 0$
7. $x^2 - x - 42 = 0$
8. $x^2 - 3x + 1 = 0$
9. $x^2 - 6x - 11 = 0$
10. $x^2 - 3x - 8 = 0$
11. $x^2 + 2x - 7 = 0$
12. $x^2 - 3x - 11 = 0$
13. $x^2 + 4x + 1 = 0$
14. $x^2 + 9x = 22$
15. $x^2 = 7x - 10$
16. $-x^2 + 13x = -30$
17. $x^2 - 21x = -20$
18. $x^2 + 8x = -15$
19. $x^2 - 6x = 0$
20. $x^2 = 11x$
21. $x^2 - 5 = 4$
22. $x^2 - 14 = -5x$
23. $11 + x^2 = -10x$
24. $3 - x = x^2$

ACTIVITY # 2

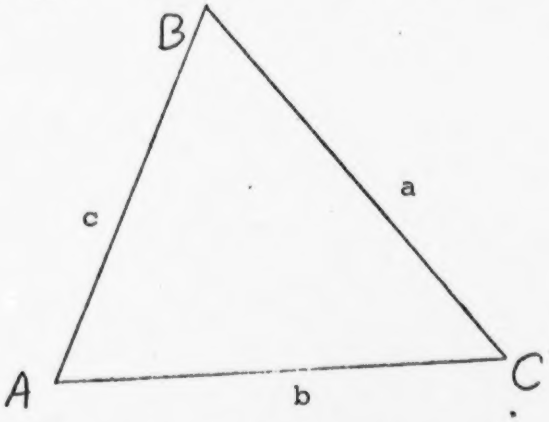
Use the Law of Cosines to solve for the unknown in the following problems. The picture is fairly accurate.

1. A = 52° b = 6.3 a = 8.4 Find c

2. A = 48° a = 11 c = 9.1 Find b

3. B = 80° a = 9 c = 7 Find b

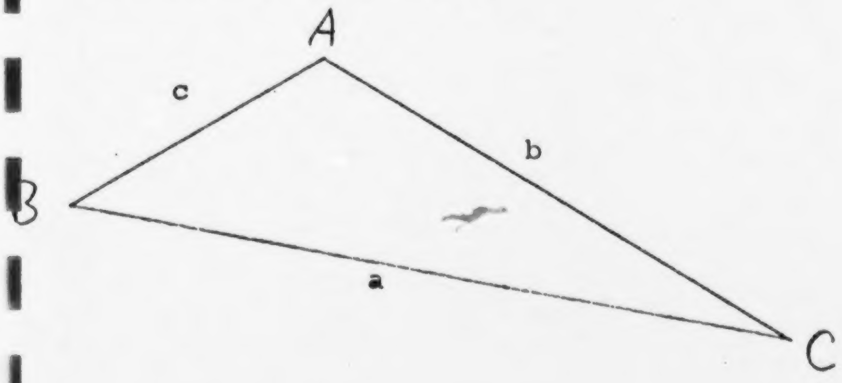
4. C = 65° a = 4 b = 9 Find c



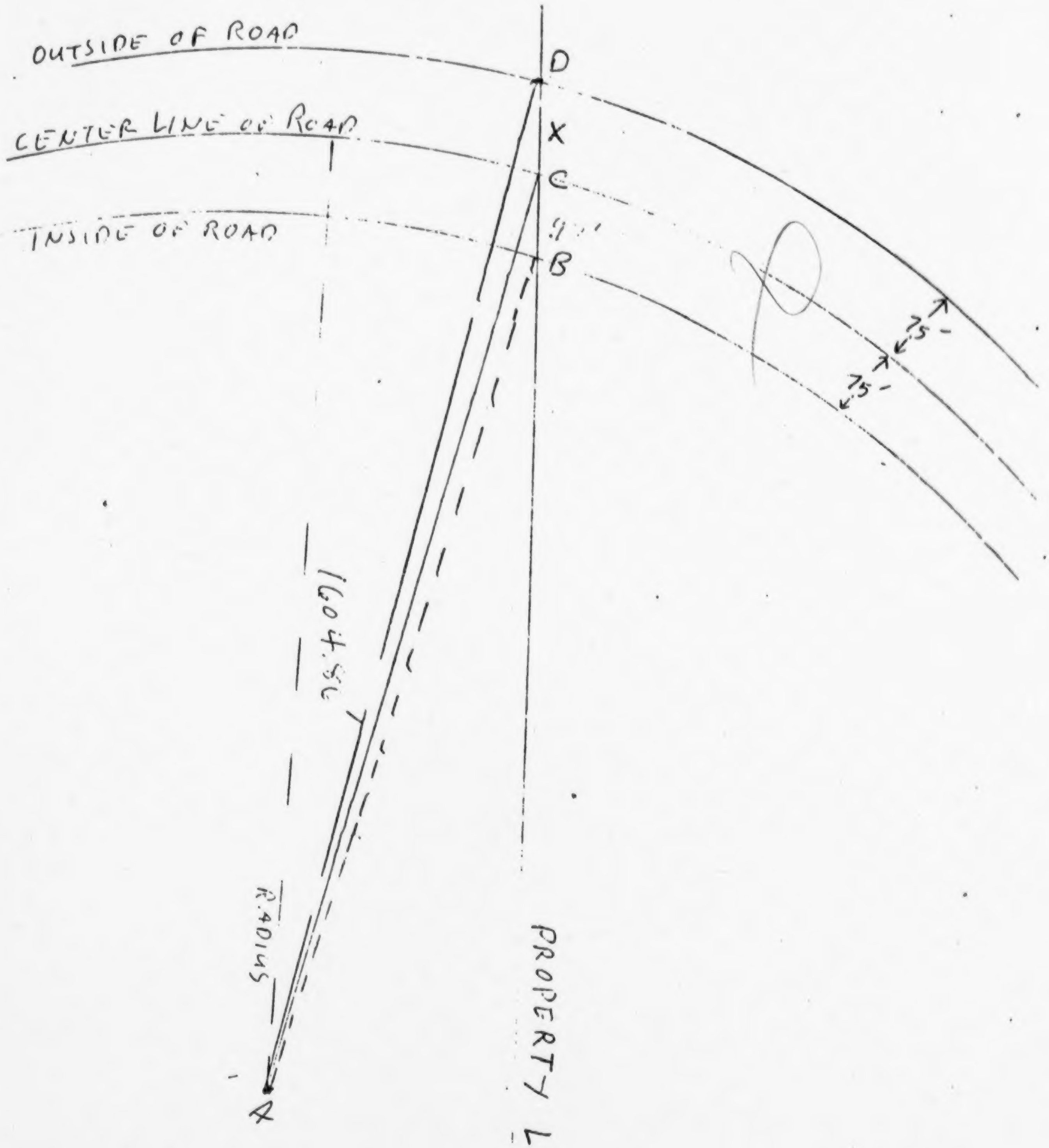
Use the triangle below for the following problem.

5. B = 35° a = 14 b = 10 Find c

The drawing is accurate.



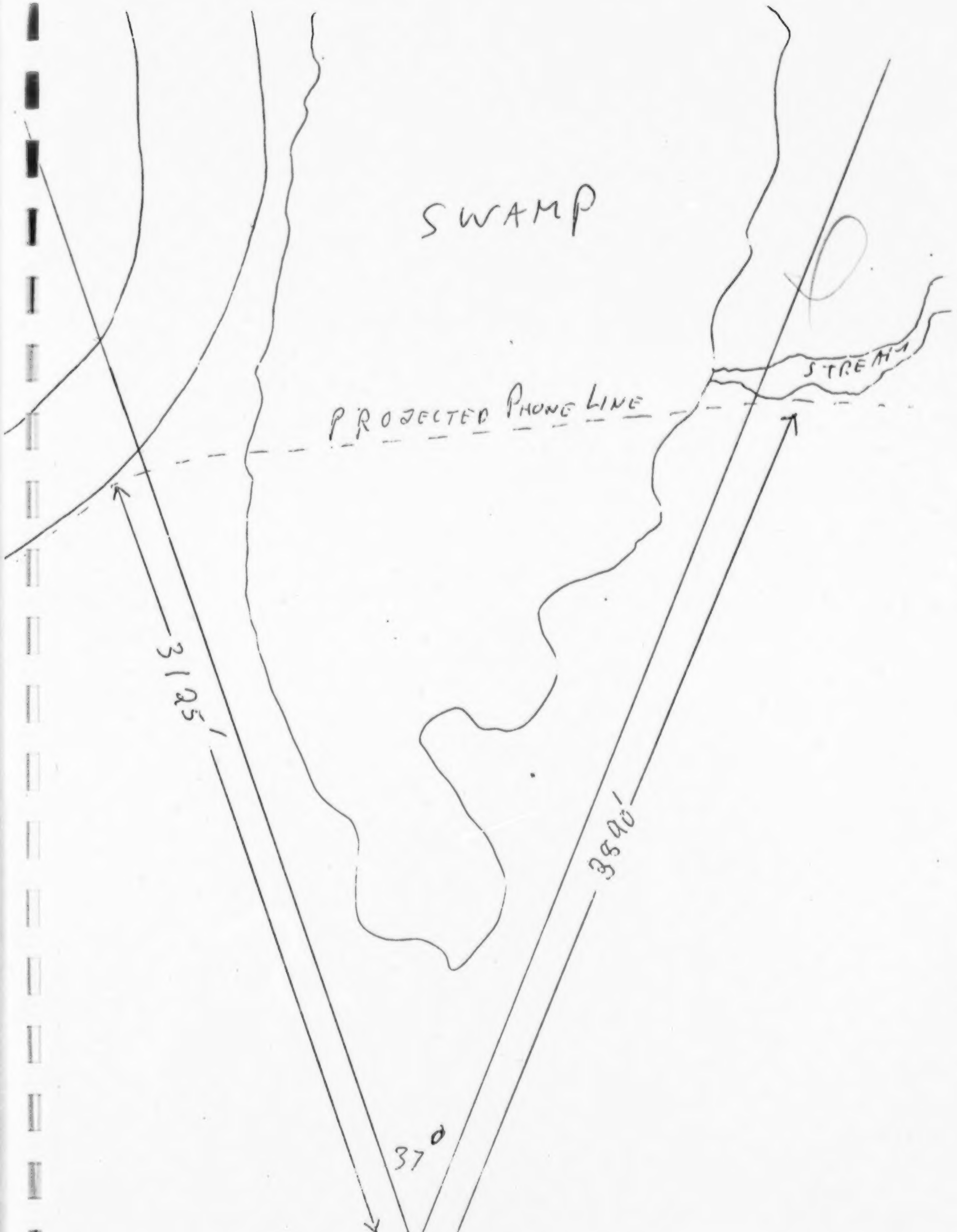
Activity # 3



Find x. Use triangles
ABC and ACD.

Post-Test

A rural phone company has to put a line across a piece of property that contains a swamp area (see picture on next page). They want to know the distance across the swamp so they can decide whether to try and cross it or go around it. The information on the picture was on the recorded deed. How far is it across the swamp?


SWAMP
STREAM
PROJECTED PHONE LINE
3125'
3820'
37°

1

Solve each equation for x. The value of x has a letter that corresponds to it in the table below. Place that letter in the blank that is numbered the same as the equation. The result is a message for you.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17

18 19 20 21 22 23 24 25 26 27 28 29

1. $3x - 4 = 23$
2. $5x + 6 = 21$
3. $5 + pqx = pq + 5$
4. $x/7 = 2$
5. $3x = 60$
6. $23x = 46$
7. $4x - 2 = 18$
8. $x/3 = 4$
9. $x + 3 = 12$
10. $x/2 = 2.5$
11. $x/11 = 2$
12. $4x - 2 = 6x - 12$
13. $2x = 18$
14. $ax = a$
15. $4x - 5 = 75$
16. $x - 6 = -1$
17. $x/25 = 4/5$
18. $3x - 6 = x + 10$
19. $\frac{x}{7} - 2 = -\frac{9}{7}$
20. $\frac{x}{6} - 3 = \frac{5}{6}$
21. $x/4 = 2$
22. $\frac{x}{2} - \frac{5}{2} = \frac{x}{3}$
23. $2x - 4 = x + 8$
24. $x/5 = 1$
25. $x/2 = 10$
26. $3x = x + 16$
27. $\frac{x}{4} = 2\frac{1}{4}$

28.

28. $\frac{2x}{7} = 4$
29. $x/7 - 5 = -4$

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

2

Solve each equation for x. The value of x has a letter that corresponds to it in the table below. Place that letter in the blank that is numbered the same as the equation. The result is a message for you. (If there are two possible answers for x, you must chooses which one is correct. If one of the answers is negative, use its absolute value to find the corresponding letter).

___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___
16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

___ ___ ___ ___ ___
32 33 34 35 36

1. $x^2 - 18x + 45 = 0$
2. $2x = 36$
3. $2x = 14 - 6$
4. $3(x - 2) = 4x - 11$
5. $ax = 18a$
6. $7/x = 14/18$
7. $28/x = 2$
8. $10/x = .5$
9. $x^2 - 16x + 64 = 0$
10. $10 - 3(x - 2) = 1$
11. $x^2 - 4x + 3 = 0$
12. $x/3 = 5$
13. $x - 21 = 0$
14. $x - 4 = 3x - 40$
15. $2(x - 11) = x - 2$
16. $x/2 = 9$
17. $x/2.5 = 6$
18. $2x + 4 = 6x - 56$
19. $(x - 1)/2 = 6$
20. $4 - (x + 2) = -6$
21. $x + 3(x + 2) = 26$
22. $x^2 - 19x + 18 = 0$
23. $3x - 2 = 13$
24. $x^2 - 7x + 12 = 0$
25. $4x - 5 = 55$
26. $x - 5 = 8$
27. $\frac{x}{6} - 2 = -\frac{7}{6}$
28. $x + 1 = \frac{2(3x - 7)}{5}$
29. $x/2 = 12 - 2$
30. $3x - 10 = x + 6$
31. $-\frac{x}{2} = -2.5$
32. $2.5x - .6 = 2.44x$
33. $x/3 = 7$
34. $3x - 5 = 7$
35. $x^3 = 343$
36. $x^2 - 9x + 20 = 0$

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

CLUSTER: ALL CLUSTERS

AREA: BACKGROUND FOR LOGICAL THINKING

TITLE: ONE + ONE = ONE

Rationale:

This package introduces a few of the ways of drawing logic diagrams. It also introduces a new algebra where $x + x = x$, and $x + 1 = 1$. This is Boolean algebra, the algebra of computers and logic.

It is hoped that after completing this package, you will have a better understanding of why different algebras are invented and why you have to be familiar with the laws that define any algebra.

Behavioral Objective:

Given a Boolean expression, you will be able to simplify it, if possible, using the postulates and theorems of Boolean algebra and then draw a schematic of the expression, with 80% accuracy.

Pre-test and Post test.

Simplify.

1. $(XY + ABC)(XY + \overline{A} + \overline{B} + \overline{C})$

2. $AB + (\overline{A}B + \overline{A}C)$

3. $\overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}CD + AB\overline{C}\overline{D} + AB\overline{C}D + A\overline{B}CD$

4. $(x + y)(x + y + z)(y + z)$

5. $ABC + AB\overline{C}D + A\overline{B}CD\overline{E} + AC$

6. $(\overline{C} + H)(\overline{A} + L)(\overline{H} + \overline{L} + O)\overline{O}C$

7. $(\overline{I} + \overline{M} + C)(\overline{C}I)$

8. Draw a logic circuit for # 1 above, before and after.

9. Draw a before and after for # 6

10. Prove your before and after of # 2 are equivalent by use of truth tables.

Information Sources:

Data Brief # 1	"And, Or circuits"
Data Brief # 2	"Not"
Data Brief # 3	"Binary truth tables"
Data Brief # 4	"Laws and theorems"
Data Brief # 5	"Circuitry"

Data Brief # 1

The condition AND is expressed by the symbol • (the sign of multiplication.)

The condition OR, by the symbol + (the sign of addition).

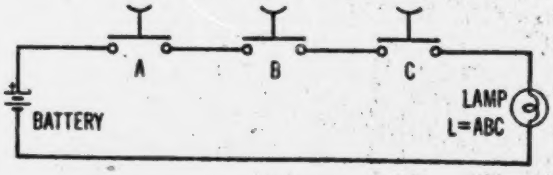
$A \cdot B + C$ is read A and B or C

$AB + C$ is read A and B or C

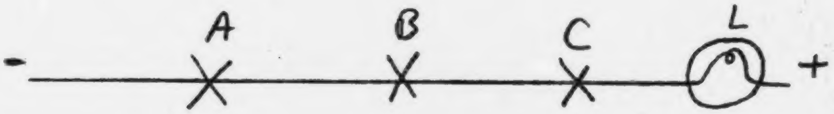
$A(B + C)$ is read A and the quantity B or C

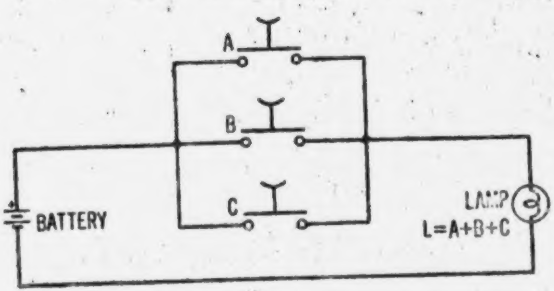
As in normal mathematics and algebra, you multiply before you add.

The AND relation is illustrated by the following electrical circuit. Examination of this circuit reveals that current from the battery mulst flow through all three push-button switches to reach light bulb L. The light is therefore lit only when A and B and C are actuated. This is expressed by the Boolean equation: L = ABC.



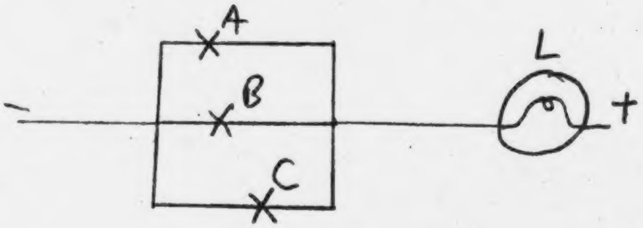
Swicth contacts ca also be represented by the symbol —✕— . Thus the circuit below also represents L = ABC.





In the OR circuit shown above, current from the battery can reach the lamp if any one (or more) of the switches is actuated. The lamp therefore can be turned on by actuating A or B or C. This is expressed by the Boolean equation: L = A + B + C.

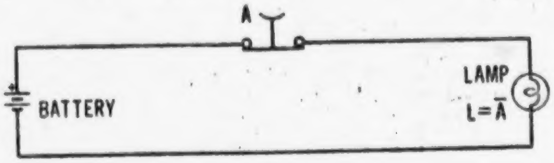
In the picture below, L = A + B + C.



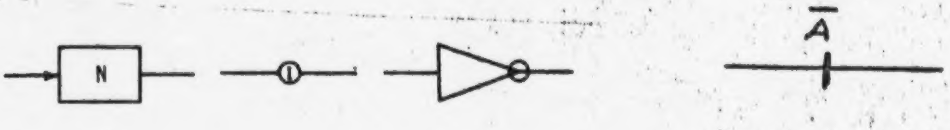
p-B

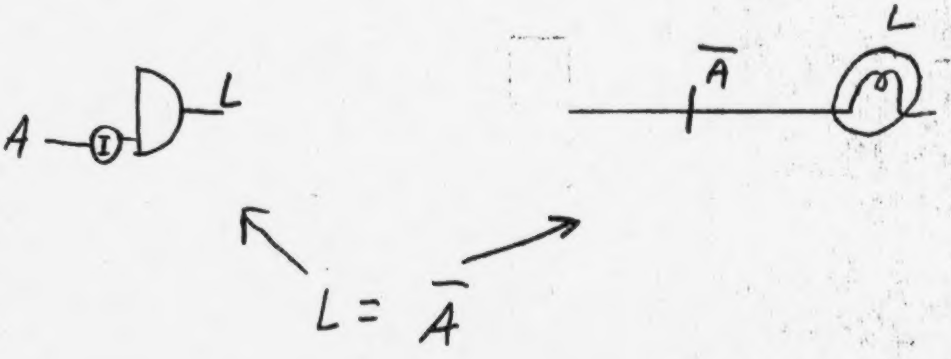
Data Brief # 2

The NOT concept is illustrated by the electrical circuit below. Because switch A has a normally closed contact, actuating the switch will break the circuit and turn off the lamp. The lamp is on only when the switch is NOT actuated. This is expressed by the Boolean equation $L = \overline{A}$.



Electronic circuits which perform the NOT function are designed so that the output terminal is at the voltage level that represents binary 1 when the input terminal is at the binary 0 level. Also, the output level is binary 1 0 when the input level is binary 1. Such circuits are also known as inverters. Some commonly used schematic symbols are shown below.





Data Brief # 3

The operation of the AND circuit is summarized in Table 1. Another way of describing this circuit is to concentrate on the condition of the two contacts and the condition of the resulting path through both of them in series. If we let 0 stand for an interrupted (open) contact or path and 1 stand for a completed (closed) path, the table of combinations in Table 2 lists all possible combinations of conditions controlled by the two switches.

Table 1.

Switch A	Switch B	Light
released	released	off
released	operated	off
operated	released	off
operated	operated	on

Table 2

Path through switch A	Path through switch B	Path through the switches in series
0	0	0
0	1	0
1	0	0
1	1	1

As in the case of the AND circuit, the OR circuit can be summarized by two tables.

Table 1

Switch A	Switch B	Light
released	released	off
released	operated	on
operated	released	on
operated	operated	on

Table 2

Path through switch A	Path through switch B	Path thrpugh the switches in parallel
0	0	0
0	1	1
1	0	1
1	1	1

Below is a proof that $AB + \overline{B} = A + \overline{B}$

A	B	AB	$\overline{B}$	$AB + \overline{B}$	$A + \overline{B}$
1	1	1	0	1	1
1	0	0	1	1	1
0	1	0	0	0	0
0	0	0	1	1	1

The two columns circled are equivalent, therefore the equality written above must be true.

Data Brief # 4

Below are the laws of algebra that hold for Boolean algebra.as well as the regular algebra.

Commutative laws

$A + B = B + A$

$AB = BA$

Associative laws

$(AB)C = A(BC)$

$(A + B) + C = A + (B + C)$

Distributive laws

$A(B + C) = AB + AC$

* $A + BC = (A + B)(A + C)$

* This is the distributive law of addition over multiplication, and of course it does not hold in regular algebra.

Below are some of the theorems of Boolean Algebra that you will find most useful.

$A + A = A$

$A + \overline{A} = 1$

$A + 1 = 1$

$A + 0 = A$

$A \cdot 1 = A$

$A \cdot 0 = 0$

$A \cdot A = A$

$A \cdot \overline{A} = 0$

$1 + 0 = 1$

$1 + 1 = 1$

$1 \cdot 0 = 0$

$1 \cdot 1 = 1$

Example: simplify the expression $A + AB$

$A + AB = A(1 + B)$ Distributive law

$= A(1)$ $1 + B = 1$

$= A$ $A \cdot 1 = A$

Example: Simplify $AB + A\overline{B} + CDE\overline{E}$

$AB + A\overline{B} + CDE\overline{E} = A(B + \overline{B}) + CDE\overline{E}$

$= A(1) + CDE\overline{E}$

$= A + CDE\overline{E}$

$= A + CD \cdot 0$

$= A + 0$

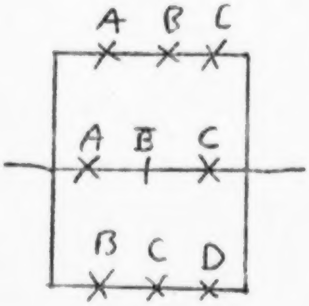
$= A$

Data Brief # 5

Below is a diagram for problem # 19, activity # 4.

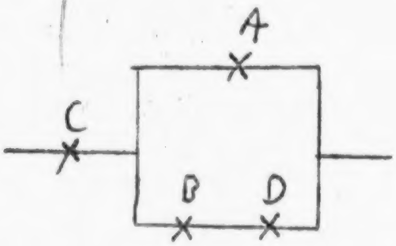
Before simplifying

$ABC + A\bar{B}C + BCD$



After simplifying

$C(A + BD)$



Activity # 1

Match each figure below to its proper equation.

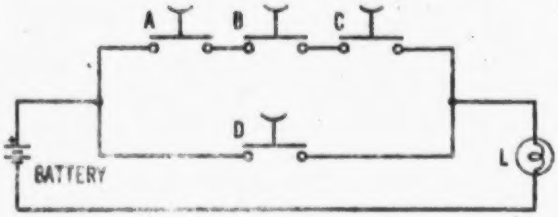


Fig. 1

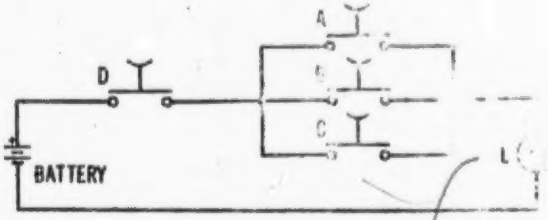


Fig. 2

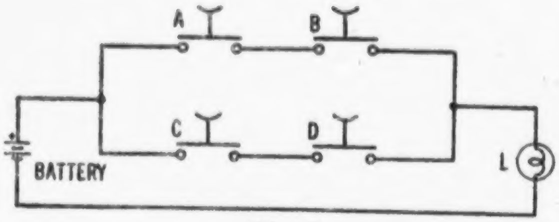


Fig. 3

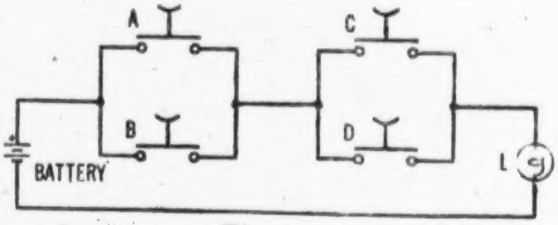


Fig. 4

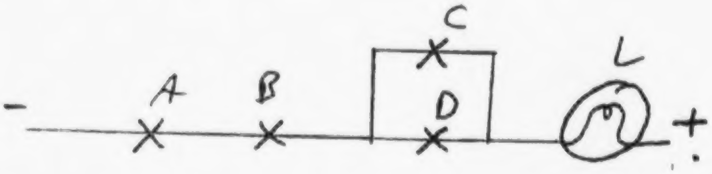


Fig. 5

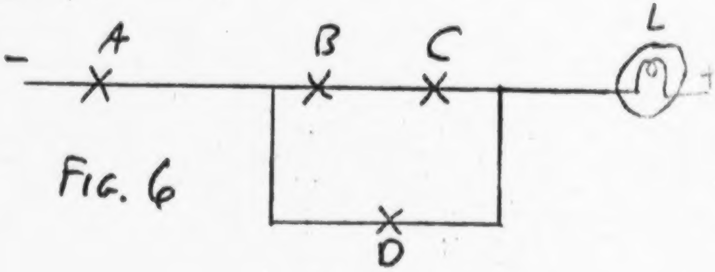


Fig. 6

Equations:

I. $L = D(A + B + C)$

II. $L = AB(C + D)$

III. $L = AB + CD$

IV. $L = ABC + D$

V. $L = A(BC + D)$

VI $L = (A + B)(C + D)$

Equation	Figure
I	
II	
III	
IV	
V	
VI	

Activity # 2

Match the number of the equation to the number of the corresponding diagram.

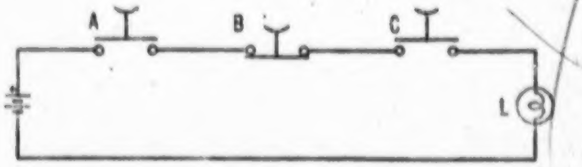


Fig. 1

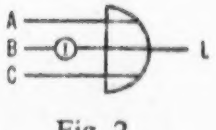


Fig. 2

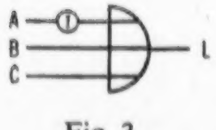


Fig. 3

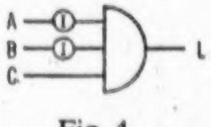


Fig. 4

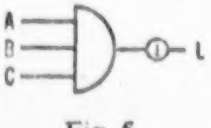


Fig. 5

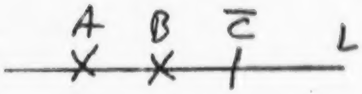


FIG. 6

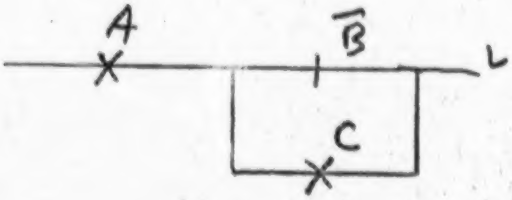


FIG. 7

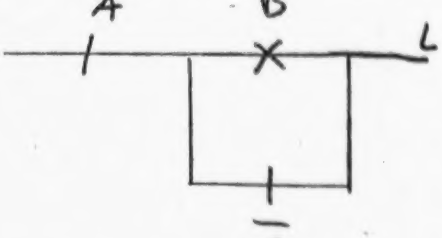
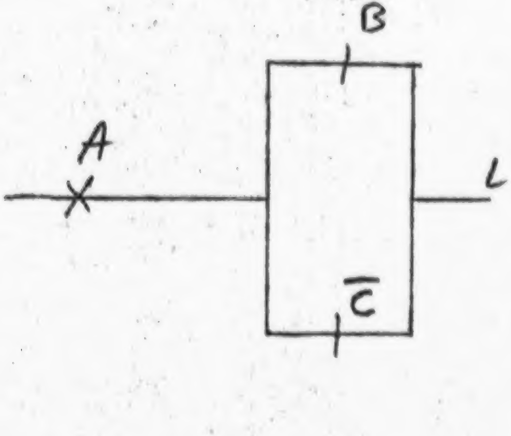


FIG. 8



continued on the following page.

ACT 2 P.2

Equations:		Figures:
I	$L = \overline{A}\overline{B}C$	
II	$L = AB\overline{C}$	
III	$L = \overline{A} + B + C$	
IV	$L = A(\overline{B} + \overline{C})$	
V	$L = A\overline{B}C$	
VI	$L = \overline{A}(B + \overline{C})$	
VII	$L = A\overline{B}C$	
VIII	$L = A + \overline{B} + C$	

The following material has been deleted: Mathmatical Truth tables.

5. $\overline{AB} = \overline{A} + \overline{B}$

6. $\overline{\overline{A}} = A$

7. $\overline{A + B} = \overline{A}\overline{B}$

8. $\overline{C}D + C = D + C$

9. $(A)(0) = 0$

10. $A(1) = A$

11. $(A + B)(A + B) = A + B$

12. $AB + AC + B\overline{C} = AC + B\overline{C}$

Activity # 4

Simplify each expression

1. $1 + 1 + 0$
2. $1 \cdot 1 \cdot A$
3. $M \cdot \overline{M} \cdot 1$
4. $X \cdot 0 + 1$
5. $C \cdot 1 + \overline{D}D$
6. $A + 0 + A + 0$
7. $A + B + 1$
8. $1(E + \overline{E})$
9. $H + H + H + \overline{H}$
10. $1 \cdot 0 \cdot A$

11. $A + \overline{A} + B$
12. $A + B + \overline{A} + AB$
13. $AA + BC + 0$
14. $AB + CDD + BD + 1$
15. $A + B + A + B + C$
16. $EF + E\overline{F}$
17. $B + BC$
18. $DE + DEF + DEG$
19. $ABC + \overline{A}BC + BCD$
20. $\overline{A}(A + B) + C$
21. $(A + B)(A + B)(A + B)$
22. $C(A + \overline{B}) + \overline{C}A + (C + \overline{B})\overline{C}$
23. $(A + AB)(A + \overline{B})$
24. $A(A + B)(A + B + C)(A + B + C + D)$
25. $(AB + BC)(A + \overline{B}\overline{C})$
26. $(L + W)(H + \overline{W})(\overline{H} + \overline{C})(R + C)(\overline{L} + \overline{C})$
27. $(\overline{D}R + R\overline{S} + \overline{B}R + \overline{B}SD)(\overline{S} + BR)$

Activity # 5

Draw a schematic (diagram) for each problem 11 -27 on activity # 4. Draw a before and an after.

CLUSTER: ALL CLUSTERS

AREA: DATA INTERPRETATION

TITLE: WHAT'D HE SAY?

Rationale:

To judge a simple statement as being either true or false, you ana lyze the one or two thoughts presented and then reach a conclusion about the statements.

But everybody doesn't speak in simple statements. A series of complicated statements can be reduced to an algebraic phrase and this phrase can be more easily studied if it is translated into a circuit diagram. The analyzing of a circuit diagram is fairly simple and can be used to test validiity.

Behavioral Objective:

Given a complicated series of statements that can be expressed using boolean algebra statements, you will be able to change the statements to algebraic expressions and simplify them, and draw a circuit picturing the simplifcation and test it for validity. Your drawing will be graded and corrected before you test for validity. You must set up and answer correctly 80% of the problems.

(You will not be told your grade on the pre-test, unless you pass, nor will you be able to look at it until this package is completed.)

PRE-TEST:

The Tardy Bus Problem

Given the following three statements as premises--

(1) If bill takes the bus, then Bill misses his appointment, if the bus is late.
(2) Bill shouldn't go home, if (a) Bill misses his appointment, and (b) Bill feels downcast.
(3) If Bill doesn't get the job, then (a) Bill feels downcast, and (b) Bill should go home.

	Is it valid to conclude--	Yes	No
Q1	That if Bill takes the bus, then Bill does get the job, if the bus is late?	___	___
Q2	That Bill does get the job, if (a) Bill misses his appointment, and (b) Bill should go home?	___	___
Q3	That if the bus is late, then (a) Bill doesn't miss his appointment, if (b) Bill doesn't ge the job?	___	___
Q4	That Bill doesn't take the bus, if (a) the bus is late, and (b) Bill doesn't get the job?	___	___
Q5	That if Bill doens't miss his appointment, then (a) Bill shouldn't go home, and (b) Bill doesn't get the job?	___	___
Q6	That Bill feels downcast, if (a) the bus is late, or (b) Bill misses his appointment?	___	___
Q7	That if Bill does get the job, then (a) Bill doesn't feel downcast, or (b) Bill shouldn't go home?	___	___
Q8	That if (a) Bill should go home, and Bill takes the bus, then (b) Bill doesn't feel downcast, if the bus is late?	___	___

Information Sources:

Data Brief # 1	"or, and, if then, etc."
Data Brief # 2	"Words to symbols"
Data Brief # 3	"Symbols to schematic"
Data Brief # 4	"Boolean Algebra"
Data Brief # 5	"Inconsistency and validity"

Data Brief # 1

"I have an apple or a bannana"can be expressed in symbols in the following ways:

A	I have an apple
B	I have a bannana

A or B

$A + B$

$A \cup B$

$A \vee B$

A truth table, using "T" and "F" instead of "1" and "0", for an OR statement is as follows:

A	OR	B
T	T	T
F	T	T
T	T	F
F	F	F

I have an apple and a bannana expressed in symbols is

A and B

$A \cdot B$

$A \cap B$

$A \wedge B$

The truth table for an AND statement is:

A	AND	B
T	T	T
F	F	T
T	F	F
F	F	F

The conditional or if-then statement is not quite so easy to understand. Written in symbols -

$A \rightarrow B$

$A \supset B$

A truth table for the statement; "If I have an Orange then I have a Citrus fruit."

If O then C

O I have an orange
C I have a citrus fruit

O	→	C
T		T
F		T
T		F
F		F

First case: If O is true and C is true is it true that If I have an orange then I have a citrus fruit? Yes.

Second Case: IF O is false and C is true, is it true that if I have an orange then I have a citrus fruit? Yes.

Third case: If O is true and C is false, is it true that if I have an orange then I have a citrus fruit? No!!!

Fourth case: If O is false and C is false, is it true that if I have an orange then I have a citrus fruit? Yes

In each case you are examining the statement "if I have an orange then I have a citrus fruit." There is only one case, the third, where this can't be true

The truth table for an if-then statement.

O	→	C
T	T	T
F	T	T
T	F	F
F	T	F

The negation symbols are as follows:

-A

$\bar{A}$

A´

~A

If A is defined as		then		has to be
	T		$\bar{A}$	F
	F			T

If X is defined as		then		has to be
	T		$\bar{X}$	F
	F			T
	T			F
	T			F

Given that

A	B
T	T
F	T
T	F
F	F

The truth table for $\bar{A} + B$ is

$\bar{A}$	+	B
F	T	T
T	T	T
F	F	F
T	T	F

Compare this truth table with the truth table for $A \rightarrow B$. This means that $A \rightarrow B \equiv \bar{A} + B$. These two expressions have to be equivalent, there truth tables match.

$\overline{A + B}$ means that the whole phrase $A + B$ is negated, not just each individual letter. To find the truth table for $\overline{A + B}$, first find the truth table for $A + B$ and then negate it.

A + B	$\overline{A + B}$	$\bar{A}$	+	$\bar{B}$
T	F	F	F	F
T	F	T	T	F
T	F	F	T	T
F	T	T	T	T

The above tables show that $\overline{A + B} \not\equiv \bar{A} + \bar{B}$

Compare the truth tables for $\overline{A + B}$ and $\bar{A}\cdot\bar{B}$

Do you see why $\overline{A + B} \equiv \bar{A}\cdot\bar{B}$

Do you see why $\overline{A\cdot B} \equiv \bar{A} + \bar{B}$

If and only if.

The symbols for this type of statement are $A \leftrightarrow B$, $A \equiv B$ $A \Leftrightarrow B$, A IFF B.

A IFF B means, if A then B and if B then A.

By truth tables you can show that this is equivalent to $\boxed{AB + \bar{A}\bar{B}.}$

Jones is at home if and only if his car is in the garage. This means Jones is at home if his car is in the garage, and if his car is in the garage, Jones is at home.

You cannot say truthfully that "I have an orange if and only if I have a citrus fruit." It is not true that if you have a citrus fruit you have an orange. You might have a lemon.

Unless.

"I can not go to the dance unless I have some money," means, "if I have no money, then I can't go to the dance."

$\bar{D}$ unless $M \equiv \bar{M} \to \bar{D} \equiv \boxed{M + \bar{D}}$

Only if.

P only if Q, is the same as if $\overline{Q}$ then $\overline{P}$.

"I can go to the dance only if I have money" in symbolic notation would be $\overline{M} + \overline{D}$ or $\boxed{M + \overline{D}}$

Niether, nor.

Niether X nor Y means $\boxed{\overline{X} \cdot \overline{Y}}$

Exclusive OR.

Either it is raining or it is not raining. It can't be both.

Either I have an apple or a bannana, and I don't have both.

$(A + B)$ and $\overline{A \cdot B}$ $(A + B)(\overline{A} + \overline{B})$

$\boxed{A\overline{B} + \overline{A}B}$

Data Brief # 2

Below are some examples of changing several sentences into symbolic notation.

Example 1.

If the litmus paper turns red, then the solution is acid. Hence if the litmus paper turns red, then either the solution is acid or something is wrong somewhere.

R The litmus paper turns red

A The solution is acid

W Something is wrong somewhere

$$(R \rightarrow A)[R \rightarrow (A + W)]$$

The oxygen in the tube either combined with the filament to form an oxide or it vanished completely. The oxygen in the tube could not have vanished completely, Therefore the oxygen in the tube combined with the filament to form an oxide.

C the oxygen in ... to form an oxide

V the oxygen vanished completely

$$(C + V)(\overline{V})(C)$$

Data Brief # 3

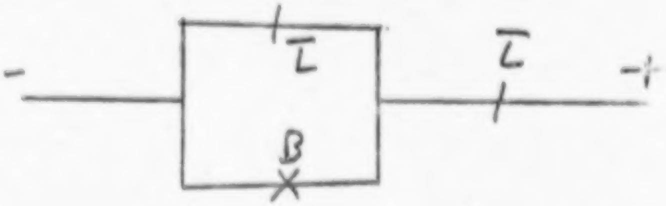
When you are to check to see if a conclusion is valid by means of looking at a diagram or schematic, DO NOT PUT THE CONCLUSION INTO THE DIAGRAM.

Example.

If your daughter Ainslee looks like Grace Kelly, she is beautiful. But you say she does not look like Grace Kelly? (Conclusion) Alas, then your daughter Ainslee is not beautiful.

$L \rightarrow B$ $\qquad (\overline{L} + B)(\overline{L})$

$\overline{L}$



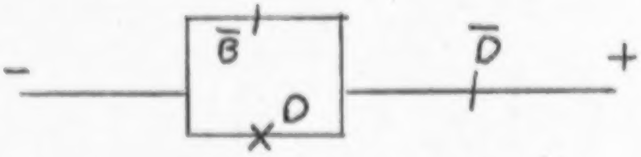
From the above diagram it can be seen that switch B can be in any position at all and there is still a path from negative to positive. Therefore the conclusion is invalid. You don't have to go thru a switch labelled $\overline{B}$.

Example.

(Conclusion) The doormouse has not buried the worm, for if he was buried the worm would be dead, and the worm is not dead.

$B \rightarrow D$

$\overline{D}$ $\qquad (\overline{B} + D)(\overline{D})$



The conclusion is valid, because the only path from negative to positive is thru the switch labelled $\overline{B}$. The reason being that you have to go thru $\overline{D}$, which means you can't go thru D.

Data Brief # 4

Do NOT put the conclusion into the diagram.

A series of statements and a conclusion are easier to prove valid or invalid if they are simplified by means of Boolean algebra first, and then a circuit is drawn of the simplification.

Example.

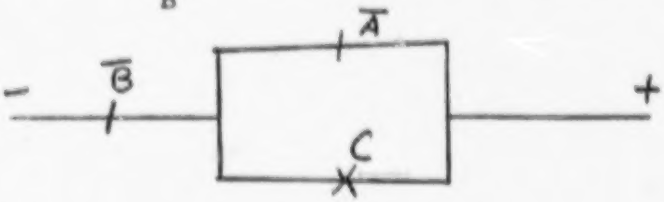
If Allen withdraws from the contest, then either Brown will win the nomination or Clark will be disappointed. Brown will not win the nomination. Therefore if Allen withdraws from the contest, Clark will be disappointed.

(A,B,C)

$A \rightarrow (B + C)$

$\overline{B}$

$$(\overline{A} + B + C)(\overline{B}) = \overline{A}\overline{B} + \overline{B}C + B\overline{B}$$
$$= \overline{B}(\overline{A} + C)$$



If A, then there is no path thru $\overline{A}$. The only path from − to + is thru C. So the conclusion is valid. To see it more readily, put your finger over the path thru $\overline{A}$. Then you can trace thru the only path left.

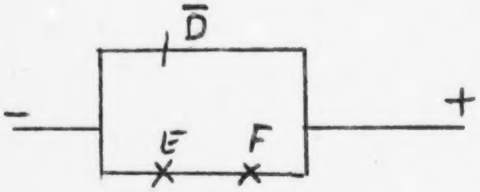
Example.

If the contract is awarded to Davis, then Edwards stands to earn a good deal of money next year. If the contract is awarded to davis, then French will suffer financial reverses. Therefore if Edwards stands to earn a great deal of money next year, French will suffer financial reverses. (D,E,F)

$D \rightarrow E$

$D \rightarrow F$

$$\begin{aligned}(\overline{D} + E)(\overline{D} + F) &= \overline{D} + \overline{D}E + \overline{D}F + EF \\ &= \overline{D}(1 + E + F) + EF \\ &= \overline{D} + EF\end{aligned}$$



Just because switch E is made does not mean you have to follow thru on that path. F could be not made and there would still be a path thru $\overline{D}$. The conclusion is invalid. With E made you are not forced to go thru F to get from - to +.

Example. The following is a simplification of problem # 16 Activity # 3, about Jacobson.

$(J \rightarrow R)(\overline{J} \rightarrow E)$

$R \rightarrow I$

$[(J \rightarrow R)(R \rightarrow I)] \rightarrow [JI + (\overline{J}\,\overline{I})]$

$JI \rightarrow T$

$JI \rightarrow D$

Below are the same statments with all but one of the conditional statements changed to "or" statements.

$(\bar{J} + R)(J + E)$

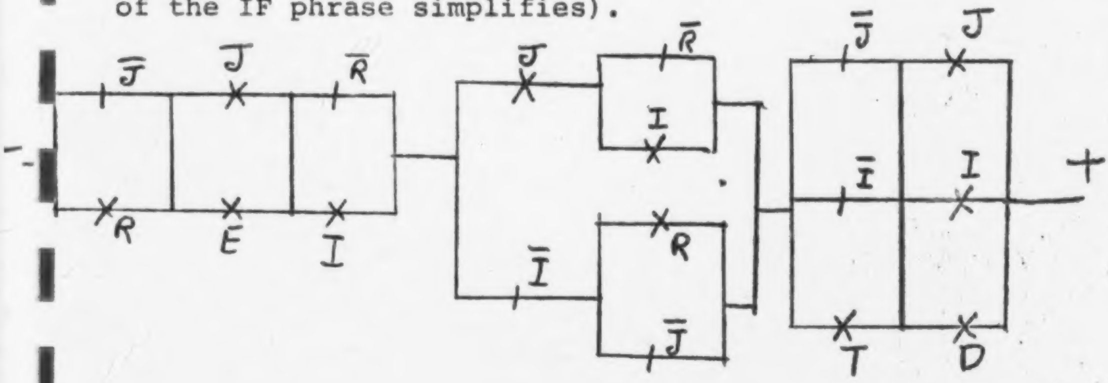
$(\bar{R} + I)$

* $[(\bar{J} + R)(\bar{R} + I)] \rightarrow (IJ + \bar{I}\bar{J}) \equiv J\bar{R} + \bar{I}R + (IJ + \bar{I}\bar{J})$

$\bar{J} + \bar{I} + T$

$J + I + D$

* This statement simplifies to either $\bar{I}J + \bar{I}R + \bar{R}J$ or to $\bar{R}J + R\bar{I}$. At this point you must see the teacher for an explanation of this simplification. (That is, the negation of the IF phrase simplifies).



Examining the above diagram, can you tell if the conclusion is valid?

The Boolean statement with the * by it can be further simplified. Before you look at the simplification on the next page, can you tell by examination of the circuit how that one section alone can be simplified?

$IJ + J\bar{R} + \bar{I}\bar{J} + \bar{I}R$

$[J\bar{R}(I + \bar{I}) + \bar{I}\bar{J} + IJ] + [\bar{I}R]$

$[\cancel{IJ\bar{R}} + \bar{I}J\bar{R} + \bar{I}\bar{J} + IJ] + [\bar{I}R]$

$\bar{I}J\bar{R} + \bar{I}\bar{J} + IJ + [(\bar{I}R)(J + \bar{J})]$

$\bar{I}J\bar{R} + \bar{I}\bar{J} + IJ + [\bar{I}JR + \cancel{\bar{I}\bar{J}}R]$

$\bar{I}J\underline{\bar{R}} + \bar{I}\bar{J} + IJ + \bar{I}JR$

$\bar{I}J(\bar{R} + R) + \bar{I}\bar{J} + IJ$

$\bar{I}J + \bar{I}\bar{J} + IJ$

$(I + \bar{I})J + \bar{I}\bar{J}$

$J + \bar{I}\bar{J}$

$J + \bar{I}$

Look at the schematic on the preceeding page and see if you agree with this simplification.

Further application of the laws of Boolean Algebra yields this result.

$IJRT + E\bar{I}\bar{J}\bar{R}D$

Looking at the above result it is very easy to see that you have to have either D or T. The conclusion is valid.

Data Brief # 5

If a set of premisses is inconsistent, any conclusion is valid. Inconsistant premisses will yield a value of zero when being simplified by Boolean algebra.

Example. Today is sunday. Today is not sunday.
Therefore the moon is made of green cheese.

$S \cdot \bar{S} \equiv 0$, so the argument is inconsistent.

Activity # 1

Express the following in logical synbolic notation.

1. Either p or not q
2. both p and not q.
3. either not or q.
4. p if and only if not q
5. It is not the case that both p and not q.
6. It is not the case that either not p or q.
7. Either p or q and not both p and q.
8. Either p or not q or both not p and q.
9. If q, then either p or q.
10. If either p or q or not r, and if not p, then either q or not r.

Activity # 2

Express the following in logical symbolic notation. Use the letters that are underlined for each phrase. State the phrase that each letter stands for.

1. Personnel should be promoted if and only if they are fully qualified.

2. If you don't talk to Marian, she won't talk to you.

3. Either Puccini or Alfano wrote the final duet of Turandot, but not both.

4. Either that bird is a loon or a great crested grebe, and if I'm mistaken, I'm not going on anymore bird-walks.

5. Unless enough neutrons are prevented from escaping from the surface of the mass, there will be no chain reaction, and without that there will be no explosion.

6. If driving a car implies having a license, then driving a horse or a car implies driving a horse or having a license.

7. If you hurry a three-toed sloth or don't feed it enough, then either it will go into a cataleptic state or lie down and die.

8. If Either Alissa did not wear her amethyst pin or Jeorme didn't leave, and Jerome did not leave, then Alissa did not wear her amethyst pin.

9. If the mouse beacon is activated, the bomber is on course, and if the cat beacon is activated, the bombs will drop. Now, either the mouse beacon is activated or the bombs will drop. The bombs won't drop. Therefore, the cat beacon is not activated and the bomber is on course.

Invent subject matter for the following expressions.

1. $\overline{P}\cdot\overline{Q}$

2. $\overline{P\cdot Q}$

3. $\overline{\overline{P}\cdot\overline{\overline{Q}}}$

4. $[P + (Q + R)] \rightarrow (S + T)$

5. $[(P + Q + R)\cdot(\overline{P}\cdot\overline{R})] \rightarrow Q$

Activity # 3

Check the validity of each conclusion by inspecting the diagram you make of the premises. DO NOT PUT THE CONCLUSION INTO THE DIAGRAM. Do not use any boolean algebra for simplification. (Use the letters designated for each phrase).

1. That orangutan is not irritated. How do I know? Listen, my friend, if an orangutan is irritated, it growls, and this orangutan is not growling. (I,G)

2. Either you are keeping a pterodactyl in the bathtub or you are meeting secretly with a cipher clerk from the embassy. It is impossible that you can be doing both. Now, we have found out that you have been meeting secretly with a cipher clerk from the embassy. It is evident, therefore, that you are not keeping a pterodactyl in the bathtub. (K,M)

3. Either she loved me or she was deceiving me. I have confirmed the fact that she loved me. So I know she was not deceiving me. (L,D)

4. It can't be that the train has passed. If the train has passed, the green flag is up, and the green flag is not up. (T,F)

5. Either it's raining or it's not raining. It's raining. Therefore, it's not raining. (R)

6. If you have the Torah, you have wisdom. Conclusion: if you have not wisdom, you have not the Torah. (T,W)

7. Either the beta particles are not penetrating the metal or the electron counter is off. The beta particles are penetrating the metal. Conclusion: the electron counter is off. (P,O)

8. Either the man's a Harvard graduate or he is not worth knowing. He is worth knowing. Therefore he is not a Harvard graduate. (G,K)

9. If you like DeKooning's pictures, you have admirable taste. If you don't like abstract art, you don't have admirable taste. Conclusion: if you don't like DeKooning's pictures, you don't like abstract art. (D,T,A)

10. Either that dufong is sick or it is nervous. If it is not nervous, it will make a good pet for the children. Conclusion: Either it's not nervous or it will make a good pet for the children. (S,N,P)

11. Either that Jaguar does not belong to Francoise or she borrowed it from Alain. If she borrowed it from Alain, she will get into trouble. Either she did not borrow it

from Alain or she won't get into trouble. Conclusion: That Jaguar belongs to Francoise. (F,A,T)

12. If the title is secure and a bank loan is withheld, we cannot go ahead with construction. We are going ahead with construction. The title is secure. Conclusion: the bank loan is not withheld. (S,W,C)

13. If Swann loves Odette, he is willing to endure the Verdurins. Either Swann is jealous or he loves Odette. If Swann is willing to endure the Verdurins, he is jealous. Either Swann is not willing to endure the Verduins or he loves Odette. Conclusion: Swann loves Odette. (L,E,J)

14. If your mother comes, I leave. If the children don't keep quiet, I leave. If I leave, your mother does not come. If the children keep quiet, then either I leave or your mother comes. Either the children are not keeping quiet or I'm not leaving. Conclusion: if the children keep quiet, then I'm leaving. (C,L,Q)

15. If and only if the priest is wearing red vestments, the Mass is for a martyr. Now we know that this Maas is for a martyr. Therefore we know that the priest is wearing red vestments. (R,M)

16. If Jacobson goes to the meeting, then a complete report will be made; but if Jacobson does not go to the meeting, then a special election will be required. If a complete report is made, then an investigation will be launched. If Jacobson's going to the meeting implies that a complete report will be made, and the making of a complete report implies that an investigation will be launched, then either Jacobson goes to the meeting and an investigation is launched or Jacobson does not go to the meeting and no investigation is launched. If Jacobson goes to the meeting and an investigation is launched, then some members will have to stand trial. But if Jacobson does not go to the meeting and no investigation is launched, then the organization will disintegrate very rapidly. Therefore either some members will have to stand trial or the organization will disintegrate very rapidly. (J - Jacobson goes to the meeting; R - A complete report is made; E - A special election is required; I- An investigation is launched; T - Some members have to stand trial; D - The organization disintegrates very rapidly.)

Activity # 4

DO NOT PUT THE CONCLUSION INTO YOUR DIAGRAM.

Draw a schematic for each problem and use it to tell if the conclusion is valid or invalid.

1. If Graham is out on the golf course, then Harvey is on duty at the hospital and Ives must have changed his policy. Harvey is not on duty at the hospital. Therefore Graham is not out on the golf course. (G,H,I)

2. If Jones discovers the plot, then if he values his life, then he will leave the country. He values his life. Therefore if Jones discovers the plot, then he will leave the country. (D,V,L)

3. If Kelly manages to borrow a car, then if he takes the expressway, then he will arrive before the deadline. Kelly will arrive before the deadline. Therefore if Kelly manages to borrow a car, then he takes the expressway. (C,E,D)

4. If Lowell is ineligible, then either Monroe is the starting fullback or Norton is the starting fullback. Monroe is not the starting full back. Therefore if Norton is not the statting fullback, then Lowell is not ineligible. (L,M,N)

5. If Olson backs the incumbent, then Peterson jumps on the bandwagon. If Peterson jumps on the bandwagon, then Quackenbush leaves the party. If Quackenbush leaves the party, then Olson does not back the incumbent. Therefore Olson does not back the incumbent. (OPQ)

6. If Robinson is nominated for the presidency, then Smith will be nominated for the vice-presidency. If Thompson is nominated for the presidency, then Smith will be nominated for the vice-presidency. Either Robinson is nominated for the presidency or Thompson is. Therefore Smith will be nominated for the vice-presidency. (R,S,T)

7. If ALice gets married, then either Betty is maid of honor or Caroline is maid of honor. If Betty is maid of honor and Caroline is maid of honor, then there will be a quarrel at the wedding. Therefore if Alice gets married, then there will be a quarrel at the wedding. (A,B,C,Q)

8. If Alice gets married, then Betty is maid of honor and Caroline is maid honor. If Either Betty is maid of honor or Caroline is maid of honor, then there will be a quarrel at the wedding. Therefore if Alice gets married, then there will be a quarrel at the wedding. (A,B,C,Q)

Simplify each problem below by using Boolean algebra and then check to see if the conclusion is valid.

9. If either George of Herbert wins, then both Jack and Kenneth lose. George wins. Therefore Jack loses. (G.H.J.K)

10. If Adams joins,then the club's social prestige will rise; and if Baker joins, then the club's financial position will be more secure. Either Adams or Baker will join. If the club's social prestige rises, then Baker will join; and if the club's financial position becomes more secure, then Wilson will join. Therefore either Baker or Wilson will join.
(A,S,B,F,W)

11. If Brown received the wire then he took the plane and if he took the plane, then he will not be late for the meeting. If the telegram was incorrectly addressed, then Brown will be late for the meeting. Either Brown received the wire or the telegram was incorrectly addressed. Therefore either Brown took the plane or he will be late for the meeting.
(R,P,L,T)

12. If rain continues, then the river rises. If rain continues and the river rises, then the bridge will wash out. If continuation of rain will wash the bridge out, then a single road is not sufficient for the town. Either a single road is sufficient for the town or the traffice engineers have made a mistake. Therefore the traffic engineers have made a mistake. (C,R,B,S,M)

13. If Ann is present, then Betty is present. If ANN and Betty are both present, then either Charlene or Doris will be elected. If either Charlene or Doris is elected, then Ethel does not really dominate the club. If Ann's presence implies that Ethel does not really dominate the club, then Florence will be the new president. So Florence will be the new president. (A,B,C,D,E,F)

14. If a man is always guided by his sense of duty, he must forego the enjoyment of many pleasures; and if he is always guided by his desire for pleasure, he must often neglect his duty. A man is either always guided by his sense of duty or always guided by hiw desire for pleasure. If a man is always guided by his sense of duty, he does not often neglect his duty; and if he is always guided by his desire for pleasure, he does not forego the enjoyment of many pleasures. Therefore a man must forego the enjoyment of many pleasures if and only if he does not often neglect his duty. (D,F,P,N)

15. The husband is wealthy and his bride is poor but honest. If a bride is poor and her husband is wealthy, then either she has made a good match, or else they will be childless or will have family trouble. She did not make a good matbh, yet they are niether quarrelsome nor have they any family trouble. Therefore they are childless. (W.P.H.G.C.F.Q)

Activity # 5

For each of the following, prove that each conclusion is either valed or invalid.

1. If the linguistics investigators are correct, then if more than one dialect was present in Ancient Greece, then different tribes came down at different times from the North. If different tribes came down at different times from the North, they must have came from the Danube River valley. But archeological excavations would have revealed traces of different tribes there if different tribes had come down at different times from the North, and archeological excavations have revealed no such trqaces there. Hence, if more than one dialect was present in Ancient Greece, then the linguistics investigators are not correct. (C,M,D,V,A)

2. If there are ordinary symptoms of a cold and the patient has a high temperature, then if there are tiny spots on his skin, he has the measles. Of course the patient cannot have measles if his record shows that he has had them before, The patient does have a high temperature and his record shows that he has had measles before. Besides the ordinary symptoms of a cold, there are tiny spots on his skin. I conclude that the patient has a virus infection. (O,T,S,M,R,V)

3. If God were willing to prevent evil, but unable to do so, he would be impotent; if he were able to prevent evil, but unwilling to do so, he would be malevolent. Evil can exist only if God is either unwilling or unable to prevent it. There is evil. If God exists, he isneither impotent nor malevolent, therefore God does not exist. (W,A,I,M,E,G)

4. If I buy a new car this spring or have my old car fixed, then I'll get up to Canada this summer and stop off in Duluth. I'll visit my parents if I stop off in Duluth. If I visit my parents, they'll insist upon my spending the summer with them. If they insist upon my spending the summer with them, I'll be there till autumn. But if I stay there till autumn, then I won't get to Canada after all! So I won't have my old car fixed. (N,F,C,D,V,I,A)

5. If Smith is intelligent and studies hard, then he will get good grades and pass his courses. If Smith studies hard but lacks intelligence, then his efforts will be appreciated; and if his efforts are appreciated, then he will pass his xourses. If Smith is intelligent, then he studies hard. Therefore Smith will pass his courses. (I,S,G,P,A)

6. If there is a single norm for greatness of poetry, then Milton and Edgar Guest cannor both be great poets. If either Pope or Dryden is regarded as a great poet, then Wordsworth is certainly no great poet; but if Wordsworth is no great poet, then niether is Keats nor Shelley. But

after all, even though Edgar Guest is not, Dryden and Keats are both great poets. Hence ther is no single norm for greatness of poetry. (N,M,G,P,D,W,K,S)

7. If the butler was present, he would have been seen; and if he was seen he would have been questioned. If he had been questioned, he would have replied; and ff he had replied, he would have been heard. But the butle was not heard. If the butler was niether seen nor heard, then he must have been on duty; and if he was on duty, he must have been present. Therefore the butler was questioned. (P,S,Q,R,H,D)

8. If the butler told the truth, then the window was closed when he entered the room; and if the gardener told the truth, then the automatic sprinkler system was not operating on the evening of the murder. If the butler and the gardener are both lying, then a conspiracy must exist to protect someone in the house and there would have been a little pool of water on the floor just inside the window. We know that the window could not have been closed when the butler entered the room. There was a little pool of water on the floor just inside the window. So in ther is a conspiracy to protect comeone in the house, then the gardener did not tell the truth.(B,W,G,S,C,P)

9. Their chief would leave the country if he feared capture, and he would not leave the country unless he feared capture. If he feared capture and left the country, the enemy's espionage network would be demoralized and powerless to harm us. If he did not fear capture and remained in the country, it would mean that he was ignorant of our own agents' work. If he really is ignorant of our agents' work, then our agents can consolidate their positions within the enemy's organization; and if our agents can consolidate their positions there, they will render the enemy's espionage network powerless to harm us. Therefore the enemy's espionage network will be powerless to harm us. (L,F,D,P,I,C)

10. If the investigators of extrasensory perception are regarded as honest, then considerable evidence for ESP must be admitted; and the doctrine of clairvoyance must be considered seriously if ESP is tentatively accepted as fact. IF considerable evidence for ESP is admitted, then it must be tentatively accepted as fact and an effort must be made to explain it. The doctrine of clairvoyance must be considered seriously if we are prepared to take seriously that class of phenomena called the occult; and if we are prepared to take seriously that class of phenomena called the occult, a new respect must be paid mediums. If we pursue the matter further, then if a new respect must be paid to mediums, we must take seriously their claims to communicate with the dead. We do pursue the matter further, but still we are practically committed to believing in ghosts if we take seriously the

mediums' claims to communicate with the dead. Hence if the investigators of ESP are regarded as honest, we are practically committed to believeing in ghosts. (H,A,C,F,E,O,M,P,D,G)

11. If we buy a lot then we will build a house. If we buy a lot then if we build a house then we will buy furniture. If we build a house then if we buy furniture then we will buy dishes. Therefore if we buy a lot then we will buy dishes. (L,H,F,D)

12. If your prices are low then your sales will be high, and if you sell quality merchandise then your customers will be satisfied. So if your prices are low and you sell quality merchandise, then your sales will be high and your customers satisfied. (L,H,Q,S)

13. If your prices are low then your sales will be high, and if you sell quality merchandise then your customers will be satisfied. So if either your prices are low or you sell quality merchandise, then either your sales will be high or your customers will be satisfied. (L.H.Q.S)

14. If Jordan joins the alliance then either Algeria or Syria boycotts it. If Kuwait joins the alliance then either Syria or Iraq boycotts it. Syria does not boycott it. Therefore if niether Algeria nor Iraq boycotts it then niether Jordan nor Kuwait joins the alliance. (J,A,S,K,I)

15. If either Jordan or Algeria joins the alliance then if either Syria or Kuwait boycotts it then although Iraq does not boycott it Yemen boycotts it. If either Iraq or Morocco does not boycott it then Egypt will join the alliance. Therefore if Jordan joins the alliance then if Syria boycotts it then Egypt will join the alliance. (J,A,S,K,I,Y,M,E)

Post- test.

Given the follwoing four statements as premises -

(1) (a) If Smith wins the nomination, then Smith feels happy and (b) Smith is not a good campaigner, if Smith feels happy.

(2) Smith loses the confidence of the party, if Smith does not win the nomination.

(3) If Smith is not a good campaigner, then Smith should resign from the party.

(4) Smith is not a good campaigner, if Smith loses the confidence of the party.

Is it valid to conclude -

________ Q1 That if Smith wins the nomination, then Smith should resign from the party?

________ Q2 That if Smith wins the nomination, and Smith does not lose the confidence of the party, then it is not so that Smith should resign from the party?

________ Q3 That Smith should resign from the party, if Smith feels happy?

________ Q4 That if Smith does not lose the confidence of the party, or Smith wins the nomination, then it is not so that Smith should resign from the party?

________ Q5 That if Smith is a good campaigner, then Smith should resign from the party?

________ Q6 That Smith should resign from the party, if Smith does not lose the confidence of the party?

________ Q7 That Smith does not win the nomination, if Smith is a good campaigner?

________ Q8 That if Smith is a good campaigner, then (a) Smith feels happy, or (b) Smith loses the confidence of his party?

________ Q9 That if Smith wins the nomination, or Smith is a good campaigner, then it is not so that Smith should resign from the party?

________ Q10 That if Smith is a good campaigner, or Smith does not lose the confidence of the party, then it is not so that Smith should resign from the party?